ECE309

INTRODUCTION TO THERMODYNAMICS & HEAT TRANSFER

10 August 2005

Final Examination R. Culham & M. Bahrami

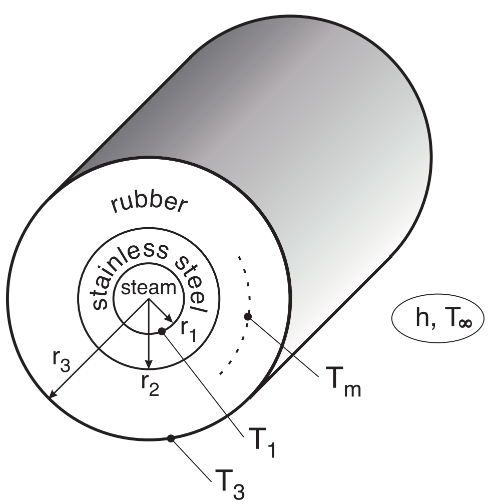
- This is a 2 - 1/2 hour, closed-book examination.
- You are permitted to use one 8.5 in.$\times$ 11 in. crib sheet (both sides), Conversion Factors (inside cover of text) and the Property Tables and Figures from your text book.
- There are 5 questions to be answered. Read the questions very carefully.
- Clearly state all assumptions.
- It is your responsibility to write clearly and legibly.
- When using correlations, it is your responsibility to verify that all limiting conditions are satisfied.

Question 1 (12 marks)

Steam flows through a stainless steel pipe and maintains the temperature of the inside surface of the pipe at $T_1 = 100\ °C$. The pipe is enclosed in a layer of rubber as shown in the sketch. Because of the imperfect contact between the rubber and the stainless steel, a thermal contact resistance must be considered at this interface. The dimensions are $r_1 = 25\ mm$, $r_2 = 35\ mm$ and $r_3 = 55\ mm$. The assembly is exposed to the environment at T_∞ and the corresponding surface heat transfer coefficient is $h = 12\ W/(m^2 \cdot K)$. The conductivity of the stainless steel is $k_{ss} = 15\ W/(m \cdot K)$ and the conductivity of the rubber is $k_{rub} = 0.15\ W/(m \cdot K)$.

Two thermocouples provide two temperature measurements - one at the surface (at $r = r_3$) where $T_3 = 40\ °C$ and the other embedded half way through the rubber (at $r = (r_2 + r_3)/2$) where $T_m = 65\ °C$. Determine:

a) the rate of heat loss per unit length of the pipe, $[W/m]$
b) the temperature of the environment, $[°C]$
c) the specific thermal contact resistance between the pipe and the rubber, $[K \cdot m^2/W]$



Part a)

Since we need to find the midpoint temperature in the rubber, break the rubber into two concentric rings each with an equivalent radius.

$$R_{total} = R_{ss} + R_c + R_{rub1} + R_{rub2} + R_{conv}$$

$$R_{ss} = \frac{\ln(r_2 - r_1)}{2\pi L k_{ss}} = \frac{\ln(0.035/0.025)}{2\pi L(m) \times 15\ (W/m \cdot K)} = \frac{0.00357}{L}\left(\frac{K}{W}\right)$$

4 marks

$$R_{rub1} = \frac{\ln(r_m - r_2)}{2\pi L k_{rub}} = \frac{\ln(0.045/0.035)}{2\pi L(m) \times 0.15\ (W/m \cdot K)} = \frac{0.2667}{L}\left(\frac{K}{W}\right)$$

$$R_{rub2} = \frac{\ln(r_3 - r_m)}{2\pi L k_{rub}} = \frac{\ln(0.055/0.045)}{2\pi L(m) \times 0.15\ (W/m \cdot K)} = \frac{0.2129}{L}\left(\frac{K}{W}\right)$$

$$R_{conv} = \frac{1}{hA} = \frac{1}{2\pi r_3 L h} = \frac{1}{2\pi \times 0.055(m) \times L \times 12\ (W/m^2 \cdot K)} = \frac{0.2411}{L}\left(\frac{K}{W}\right)$$

Since the heat flow is the same throughout each section of the pipe, the heat loss per unit length can be written as

$$\dot{Q}' = \frac{T_m - T_3}{R_{rub2} \times L} = \frac{(65 - 40)\ K}{\frac{0.2129}{L}\left(\frac{K}{W}\right) \times L(m)} = 117.426\ \frac{W}{m} \Leftarrow$$

1 mark

Part b)

The environmental temperature can be determined from

$$\dot{Q}' = \frac{\Delta T}{R_{conv}} = \frac{(T_3 - T_\infty)}{1/(2\pi \times r_3 \times h)} = (2\pi \times r_3 \times h) \times (T_3 - T_\infty)$$

1 mark

solving for T_∞

$$T_\infty = T_3 - \frac{\dot{Q}'}{2\pi \times r_3 \times h} = 40\,^\circ C - \frac{117.426\ W/m}{2\pi \times 0.055\ (m) \times 12\ (W/m^2 \cdot K)} = 11.69\,^\circ C \Leftarrow$$

2 marks

Part c)

Since we know temperatures at T_1 and T_m we need to find the resistance between these two points. Then the contact resistance can be written as

$$R_c = R_{1\to m} - R_{ss} - R_{rub1}$$

First,

1 mark

$$\dot{Q}' = \frac{T_1 - T_m}{R_{1\to m} \times L} \quad \Rightarrow \quad R_{1\to m} = \frac{T_1 - T_m}{\dot{Q}' \times L} = \frac{(100 - 65)\ K}{117.426\ (W/m) \times L\ (m)} = \frac{0.2981}{L}\left(\frac{K}{W}\right)$$

Therefore

$$R_c = \frac{0.2981}{L}\left(\frac{K}{W}\right) - \frac{0.00357}{L}\left(\frac{K}{W}\right) - \frac{0.2667}{L}\left(\frac{K}{W}\right) = \frac{0.02783}{L}\left(\frac{K}{W}\right)$$

1 mark

and

$$\begin{aligned} R''_c &= R_c \times A = R_c \times (2\pi \times r_2 \times L) \\ &= \frac{0.02783}{L}\left(\frac{K}{W}\right) \times 2\pi \times 0.035\ (m) \times L\ (m) \\ &= 0.0061\ \frac{K \cdot m^2}{W} \Leftarrow \end{aligned}$$

2 marks

Question 2 (12 marks)

a) As the length of a fin increases, would you expect i) the fin efficiency and ii) the fin effectiveness to increase or decrease? Explain. (3 marks)

b) Oil is driven over a heated surface where hydrodynamic and thermal boundary layers are formed. The flow is laminar. What fluid property can be used as an indication of the relative thickness of the two boundary layers? Explain why. Which boundary layer is thicker? Why? (3 marks)

c) If 10% of the emitted radiant energy from an incandescent light bulb falls in the visible wavelength band (i.e. between 0.4 and $0.76\ \mu m$), determine the operating temperature of the filament. (4 marks)

d) How would the value of the critical radius of insulation change if radiation effects are considered? Explain. (2 marks)

Part a)

i) The fin efficiency can be written as

$$\eta = \frac{\tanh mL}{mL}$$

For a fixed value of the fin parameter, m, you will note that $(\tanh L)/L$ will always decrease as L is increased. This means that the fin efficiency will decrease for an increase in the fin length.

or

From Fig. 8-59, we see that $\xi = \left(L + \frac{1}{2}t\right)\sqrt{h/kt}$. As $L \uparrow$, the efficiency, $\eta \downarrow$. 1.5 marks

ii) The fin effectiveness is given as

$$\epsilon = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}}$$

As the fin length, L, increases, the overall heat transfer of the fin, $\dot{Q}_{fin}$ will also increase.

Therefore, the fin effectiveness will increase for an increase in the fin length. 1.5 marks

Part b)

The Prandtl number is the property that gives a clear indication of the relative thickness of the boundary layers.

$$Pr = \frac{\nu}{\alpha} = \frac{\text{rate of momentum diffusion}}{\text{rate of thermal diffusion}}$$

The Prandtl number for oil is between 84 - 47100 depending on the temperature. (Table A-18). This means that momentum is diffusing faster than heat and the hydrodynamic boundary layer will be thicker than the thermal boundary layer. 3 marks

Part c)
From the general formulation for the blackbody radiation function we can write

$$f_{\lambda_1}(T) - f_{\lambda_2}(T) = \frac{\int_{\lambda_1}^{\lambda_2} E_{b\lambda}(T)\, d\lambda}{\sigma T^4} = 0.1$$ 1 mark

Since T is unknown, we will have to iterate to find a value of T that satisfies the above equation. From Table 12.2:

at $T = 3000\ K$

$$\begin{aligned} \lambda_1 T &= 0.4\ \mu m \times 3000\ K = 1200\ \mu m \cdot K \ \Rightarrow\ f_{0\to 0.4} = 0.002134 \\ \lambda_2 T &= 0.76\ \mu m \times 3000\ K = 2280\ \mu m \cdot K \ \Rightarrow\ f_{0\to 0.7} = 0.116635 \end{aligned}$$

$$f_{0.4\to 0.76}(3000\ K) = 0.116635 - 0.002134 = 0.114501 \longleftarrow \text{ slightly high}$$

at $T = 2900\ K$ 2 marks

$$\begin{aligned} \lambda_1 T &= 0.4\ \mu m \times 2900\ K = 1160\ \mu m \cdot K \ \Rightarrow\ f_{0\to 0.4} = 0.001771 \\ \lambda_2 T &= 0.76\ \mu m \times 2900\ K = 2204\ \mu m \cdot K \ \Rightarrow\ f_{0\to 0.7} = 0.101675 \end{aligned}$$

$$f_{0.4\to 0.76}(2900\ K) = 0.101675 - 0.001771 = 0.099904 \longleftarrow\ 9.99\% \approx 10\%$$

The filament temperature would be $2900\ K$ $\Leftarrow$ 1 mark

Part d)
The critical radius of insulation can be written as:

$$r_{crit} = \frac{k}{h} \text{ cylinder} \qquad \text{and} \qquad r_{crit} = \frac{2k}{h} \text{ sphere}$$

If radiation effects are included this would result in a higher effective heat transfer coefficient such that

$$h_{eff} = h_{cond} + h_{rad} \qquad \text{where} \qquad h_{rad} = \epsilon\sigma(T_s + T_{sur})(T_s^2 + T_{sur}^2)$$

A higher effective heat transfer coefficient will always lead to a lower value of critical radius of insulation. $\Leftarrow$ 2 marks

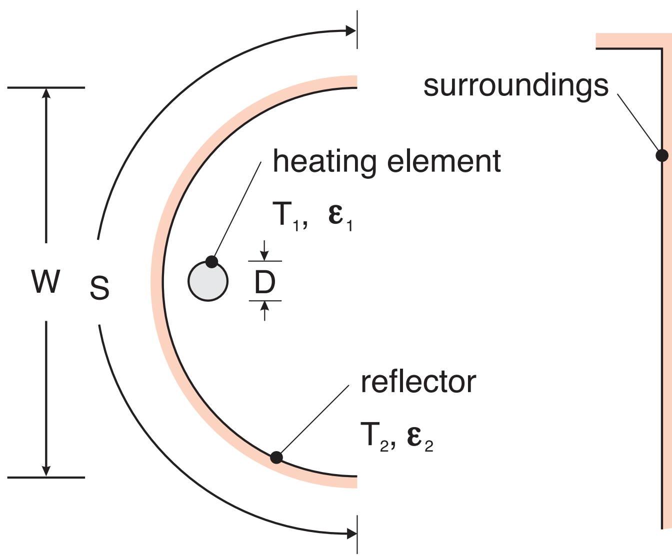
Question 3 (13 marks)

A radiant space heater consists of a long, diffuse, gray, cylindrical heating element that has a diameter of $30\ mm$ and is backed by a curved metallic reflector. The surface temperature and emissivity of the heating element are $T_1 = 945\ K$ and $\epsilon_1 = 0.8$. The temperature of the air and the temperature of the surroundings are both $300\ K$. Convection heat transfer from the element can be considered to be negligible. The reflector, which has an arc length of $S = 0.3\ m$, may be assumed to be an isothermal, diffuse, gray surface with an emissivity of $\epsilon_2 = 0.1$. The base of the reflector has a width of $W = 0.15\ m$. The steady state temperature of the reflector plate is $T_2 = 385\ K$ and the view factor of the heater with respect to the reflector is $F_{12} = 0.625$.

a) Draw the equivalent circuit to describe radiation heat transfer between the reflector, the heating element and the surroundings. Omit the portion of the circuit describing heat transfer from the back of the reflector.

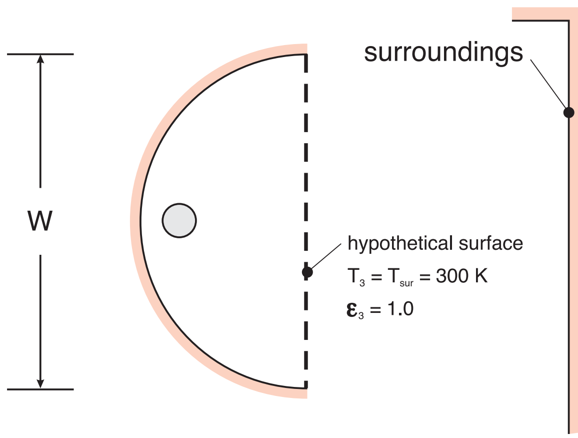
b) Evaluate all resistances and nodal potentials (per unit length of the heater).

c) Calculate the electrical power input to the heating element per unit length.

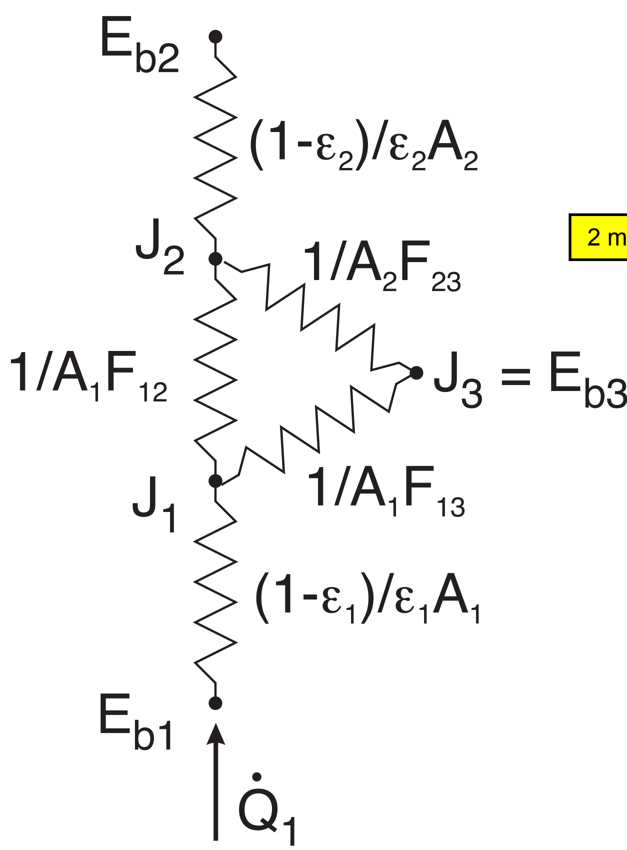


Assume:

1. surface are gray and diffuse
2. the surroundings form a large enclosure which may be represented by a hypothetical surface of temperature $T_3 = T_{sur}$ and emissivity, $\epsilon_3 = 1$



Part a)



2 marks

Part b)
The view factors are obtained as follows:

$$F_{13} = 1 - F_{12} = 1 - 0.625 = 0.375 \quad \text{(summation rule)}$$

1 mark

$$F_{32} = 1 - F_{31} = 1 - F_{13}\left(\frac{A_1}{A_3}\right) = 1 - 0.375\left(\frac{\pi D L}{W L}\right) = 1 - 0.375 \times \pi \times \frac{0.03\ m}{0.15\ m} = 0.764$$

The circuit resistances are

$$\frac{1-\epsilon_1}{\epsilon_1 A_1} = \frac{1-0.8}{0.8 \times \pi \times 0.03\ (m) \times L\ (m)} = \frac{2.65}{L}\ (m^{-2})$$

$$\frac{1}{A_1 F_{12}} = \frac{1}{\pi \times 0.03\ (m) \times L\ (m) \times 0.625} = \frac{16.98}{L}\ (m^{-2})$$

2 marks

$$\frac{1}{A_1 F_{13}} = \frac{1}{\pi \times 0.03\ (m) \times L\ (m) \times 0.375} = \frac{28.30}{L}\ (m^{-2})$$

$$\frac{1}{A_2 F_{23}} = \frac{1}{A_3 F_{32}} = \frac{1}{0.15\ (m) \times L\ (m) \times 0.764} = \frac{8.73}{L}\ (m^{-2})$$

$$\frac{1-\epsilon_2}{\epsilon_2 A_2} = \frac{1-0.1}{0.1 \times 0.30\ (m) \times L\ (m)} = \frac{30}{L}\ (m^{-2})$$

The nodal potentials are

$$E_{b1} = \sigma T_1^4 = 5.67 \times 10^{-8}\left(\frac{W}{m^2 \cdot K^4}\right) \times (945\ K)^4 = 45,218\ (W/m^2) \leftarrow$$

$$E_{b2} = \sigma T_2^4 = 5.67 \times 10^{-8}\left(\frac{W}{m^2 \cdot K^4}\right) \times (385\ K)^4 = 1,246\ (W/m^2) \leftarrow$$

1 mark

$$E_{b3} = \sigma T_3^4 = 5.67 \times 10^{-8}\left(\frac{W}{m^2 \cdot K^4}\right) \times (300\ K)^4 = 459\ (W/m^2) \leftarrow$$

Part c)
The input power can be obtained as follows

$$\dot{Q}_1 = \frac{E_{b1} - J_1}{\dfrac{1-\epsilon_1}{\epsilon_1 A_1}}$$

First we need to find the radiosity by applying radiation balances to J_1 and J_2.

$$\frac{E_{b1} - J_1}{\dfrac{1-\epsilon_1}{\epsilon_1 A_1}} = \frac{J_1 - J_2}{\dfrac{1}{A_1 F_{12}}} + \frac{J_1 - E_{b3}}{\dfrac{1}{A_1 F_{13}}}$$

2 marks

$$\frac{45,218 - J_1}{2.65/L} = \frac{J_1 - J_2}{16.98/L} + \frac{J_1 - 459}{28.30/L}$$

$$J_2 = 8.01 J_1 - 290,011.93$$

and

$$\frac{E_{b2} - J_2}{\dfrac{1-\epsilon_2}{\epsilon_2 A_2}} = \frac{J_2 - J_1}{\dfrac{1}{A_1 F_{12}}} + \frac{J_2 - E_{b3}}{\dfrac{1}{A_2 F_{23}}}$$

2 marks

$$\frac{1,246 - J_2}{30/L} = \frac{J_2 - J_1}{16.98/L} + \frac{J_2 - 459}{8.73/L}$$

$$J_1 = 3.51 J_2 - 1597.99$$

$$J_1 = 3.51(8.01 J_1 - 290,011.93) - 1597.99$$

$$J_1 = 37,600\ W/m^2 \Leftarrow$$

1 mark

Which in turn gives

$$J_2 = 8.01 J_1 - 290,011.93 = 8.01(37,600) - 290,011.93 = 11,164\ W/m^2 \Leftarrow$$

1 mark

and finally power input per unit length is

$$\frac{\dot{Q}_1}{L} = \frac{E_{b1} - J_1}{\dfrac{1-\epsilon_1}{\epsilon_1 A_1}} \times \frac{1}{L} = \frac{(45,218 - 37,600)\ W/m^2}{\dfrac{2.65}{L}(m^{-2}) \times L\ (m)} = 2,874.7\ (W/m) \Leftarrow$$

1 mark

Question 4 (10 marks)

A steel bar, initially at an ambient temperature of $25\ °C$, is heat treated by suspending it vertically in the lengthwise direction and passing it through a conveyor oven maintained at a temperature of $175\ °C$. The steel bar has dimensions of $32\ mm \times 10\ mm \times 1.1\ m$ and thermophysical properties as follows:

$$\begin{aligned}\rho &= 8131\ kg/m^3 \\ C_p &= 434\ J/(kg \cdot K) \\ k &= 41\ W/(m \cdot K) \\ \alpha &= 11.6 \times 10^{-6}\ m^2/s\end{aligned}$$

In order to achieve the proper heat treatment, the steel bar must reach a temperature of at least $140\ °C$ for $35\ min$.

If all thermophysical properties and oven conditions remain the same, how long should a $76\ mm \times 35\ mm \times 1.6\ m$ steel bar remain in the oven to achieve the proper heat treatment at $140\ °C$?

Since we do not have a value for the heat transfer coefficient, h, we will have to assume that $Bi < 0.1$ and then check later to see if the assumption is correct. First examine the smaller bar to determine the value of h.

The characteristic length can be determined as

$$\begin{aligned}\mathcal{L} = \frac{V}{A} &= \frac{0.032\ m \times 0.010\ m \times 1.1}{2(0.032\ m \times 0.010\ m) + 2(0.032\ m \times 1.1\ m) + 2(0.010\ m \times 1.1\ m)} \\ &= 0.00378\ m\end{aligned}$$

1 mark

Using a lumped capacitance procedure, we can write

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-t/\tau}$$

where

$$\begin{aligned}\tau &= \frac{mC_p}{Ah} = \frac{\rho V C_p}{hA} = \frac{\rho \mathcal{L} C_p}{h} \\ &= \frac{8131\ kg/m^3 \times 0.00378\ m \times 434\ J/(kg \cdot K)}{h\ (W/m^2 \cdot K)} \\ &= \frac{13,339.1}{h}\ (s)\end{aligned}$$

1 mark

Therefore

$$\frac{(140 - 175)\ °C}{(25 - 175)\ °C} = \exp\left(-\frac{35\ min \times 60\ s/min}{\dfrac{13,339.1}{h}\ (s)}\right)$$

1 mark

$$0.2333 = \exp(-0.1574h)$$

and

$$h = 9.25\, W/m^2 \cdot K$$ 1 mark

Check the Biot number to see if a lumped system approach was correct.

$$Bi = \frac{h\mathcal{L}}{k} = \frac{9.25\, W/m^2 \cdot K \times 0.00378\, m}{41\, W/(m \cdot K)} = 0.0009$$ 1 mark

Since $Bi << 0.1$ the lumped system approach is valid.

The characteristic length for the larger steel bar is

1 mark

$$\begin{aligned} \mathcal{L} = \frac{V}{A} &= \frac{0.076\, m \times 0.035\, m \times 1.6}{2(0.076\, m \times 0.035\, m) + 2(0.076\, m \times 1.6\, m) + 2(0.035\, m \times 1.6\, m)} \\ &= 0.0118\, m \end{aligned}$$

$$\tau = \frac{\rho \mathcal{L} C_p}{h} = \frac{8131\, kg/m^3 \times 0.0118\, m \times 434\, J/(kg \cdot K)}{9.25\, W/m^2 \cdot K} = 4501.7\, s$$ 1 mark

Therefore

$$\begin{aligned} \frac{(140 - 175)\, ^\circ C}{(25 - 175)\, ^\circ C} &= \exp\left(-\frac{t(s)}{4501.7\, (s)}\right) \\ 0.2333 &= \exp(-t/4501.7) \end{aligned}$$

2 marks

$$t = 6551.9\, s = 109.2\, minutes \Leftarrow$$

Check again to see if the Biot number is small.

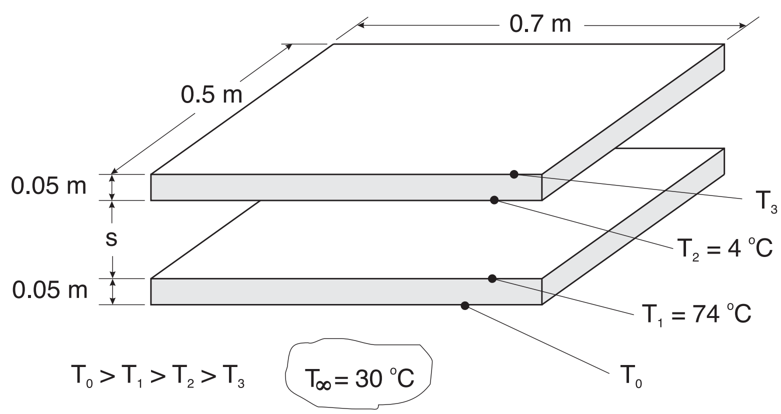
$$Bi = \frac{h\mathcal{L}}{k} = \frac{9.25\, W/m^2 \cdot K \times 0.0118\, m}{41\, W/m \cdot K} = 0.00266 \quad << 0.1$$ 1 mark

Question 5 (13 marks)

Consider two $0.5\ m \times 0.7\ m \times 0.05\ m$ horizontal, parallel plates separated by a distance 's' in a room at a constant temperature of $T_\infty = 30\ ^\circ C$. The plates are made of iron (Armco, 99% pure). The top surface of the lower plate is maintained at a temperature of $74\ ^\circ C$ while the bottom surface of the upper plate is maintained at $4\ ^\circ C$.

a) The distance between the plates, 's', is large so that the space between the plates, can be assumed to be a constant temperature at $T_\infty = 30\ ^\circ C$. Determine the net convection heat transfer rate from the inner surfaces of the plates.

b) The distance between the plates is $s = 0.1\ m$. Assume 1-D heat flow between the plates (through the air gap). Determine an equivalent effective thermal conductivity for the overall system (where the system includes the plates and the air gap. You can use the same convective heat transfer coefficients, h_1 and h_2 from part a). Neglect radiation effects.



Assume:

- steady state heat transfer
- air pressure is at 1 atm
- 1-D conduction

The properties of the air are evaluated at the film temperature:

$$T_{f1} = \frac{T_1 + T_\infty}{2} = \frac{74 + 30}{2} = 52\ ^\circ C = 325\ K$$

1 mark

$$T_{f2} = \frac{T_2 + T_\infty}{2} = \frac{4 + 30}{2} = 17\ ^\circ C = 290\ K$$

Air properties from Table A-19

1 mark

Surface 1

$$k_1 = 0.0279\ W/m \cdot K$$
$$Pr_1 = 0.709$$
$$\nu_1 = 1.815 \times 10^{-5}\ m^2/s$$
$$\beta_1 = \frac{1}{T_f} = \frac{1}{325\ K} = 0.00308\ K^{-1}$$

Surface 2

$$k_2 = 0.0253\ W/m \cdot K$$
$$Pr_2 = 0.714$$
$$\nu_2 = 1.48 \times 10^{-5}\ m^2/s$$
$$\beta_2 = \frac{1}{T_f} = \frac{1}{290\ K} = 0.00345\ K^{-1}$$

For the horizontal, hot surface facing up, the characteristic length and the Rayleigh number are:

$$\delta = \frac{A}{P} = \frac{(0.5\,m)(0.7\,m)}{2(0.5+0.7)\,m} = 0.1458\,m \quad \boxed{\text{1 mark}}$$

$$Ra_1 = \frac{g\beta_1(T_1 - T_\infty)\delta^3}{\nu_1^2} Pr_1$$

$$= \frac{\left(9.8\,\frac{m}{s^2}\right)\left(0.00308\,\frac{1}{K}\right)(74-30)\,K \times (0.1458\,m)^3}{(1.815\times 10^{-5}\,m^2/s)^2}(0.709)$$

$$= 8.86\times 10^6 \quad \boxed{\text{1 mark}}$$

From Eq. 11-11

$$Nu_1 = 0.54 Ra_1^{1/4} = 0.54(8.86\times 10^6)^{1/4} = 29.46$$

$$Nu_1 = \frac{h_1\delta}{k_1} = 29.46 \quad \longrightarrow \quad h_1 = \frac{29.45\times 0.0279\,W/m\cdot K}{0.1458\,m} = 5.64\,W/m^2\cdot K \quad \boxed{\text{1 mark}}$$

For the lower surface of a horizontal cold plate, the characteristic length and the Rayleigh number are:

$$\delta = \frac{A}{P} = \frac{(0.5\,m)(0.7\,m)}{2(0.5+0.7)\,m} = 0.1458\,m$$

$$Ra_2 = \frac{g\beta_2(T_\infty - T_2)\delta^3}{\nu_2^2} Pr_2$$

$$= \frac{\left(9.8\,\frac{m}{s^2}\right)\left(0.00345\,\frac{1}{K}\right)(30-4)\,K \times (0.1458\,m)^3}{(1.48\times 10^{-5}\,m^2/s)^2}(0.714)$$

$$= 8.88\times 10^6 \quad \boxed{\text{1 mark}}$$

From Eq. 11-11

$$Nu_2 = 0.54 Ra_2^{1/4} = 0.54(8.88\times 10^6)^{1/4} = 29.48$$

$$Nu_2 = \frac{h_2\delta}{k_2} = 29.48 \quad \longrightarrow \quad h_2 = \frac{29.48\times 0.0253\,W/m\cdot K}{0.1458\,m} = 5.12\,W/m^2\cdot K \quad \boxed{\text{1 mark}}$$

The net heat transfer can be written as

$$\dot{Q}_{net\to\infty} = \dot{Q}_1 - \dot{Q}_2$$

$$= A[h_1(T_1 - T_\infty) - h_2(T_\infty - T_2)]$$

$$= (0.5\,m \times 0.7\,m)[(5.64\,W/m^2\cdot K)(74-30)\,K - (5.12\,W/m^2\cdot K)(30-4)\,K]$$

$$= 40.264 \Leftarrow W \quad \boxed{\text{2 marks}}$$

Part b)

$$k_{iron} = 72.7\ W/m \cdot K \qquad \text{Table A-14}$$

1 mark

$$k_{air}(@312\ K) = 0.02694\ W/m \cdot K \qquad \text{Table A-19}$$

The total resistance in the system can be written as a series resistance path where

$$\begin{aligned}
R_{system} &= R_{plate_1} + R_{conv_1} + R_{air} + R_{conv_2} + R_{plate_2} \\
&= \frac{t_1}{k_1 A} + \frac{1}{h_1 A} + \frac{s}{k_{air} A} + \frac{1}{h_2 A} + \frac{t_2}{k_2 A} \\
&= \frac{1}{A}\left(\frac{0.05\ m}{72.7\ \dfrac{W}{m \cdot K}} + \frac{1}{5.64\ \dfrac{W}{m^2 \cdot K}} + \frac{0.1\ m}{0.02694\ \dfrac{W}{m \cdot K}} \right. \\
&\qquad \left. + \frac{1}{5.12\ \dfrac{W}{m^2 \cdot K}} + \frac{0.05\ m}{72.7\ \dfrac{W}{m \cdot K}} \right) \\
&= 4.0853/A
\end{aligned}$$

2 marks

The effective resistance can be written as

$$\frac{s + 2t}{k_{eff} A} = \frac{4.0853}{A} \quad \Rightarrow \quad k_{eff} = \frac{0.1\ m + 2(0.05\ m)}{4.0853} = 0.0490\ W/m \cdot K \Leftarrow$$

1 mark