Solutions for suggested problems chapter 1

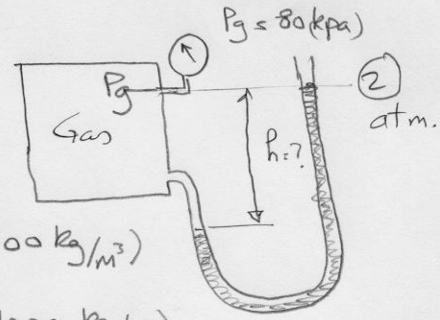
1 - Problem 1

$P_g = 80$ kpa

find $\underline{\underline{h}}$ if

a) fluid is mercury ($\rho_{Hg} = 13600\ kg/m^3$)

b) fluid is water ($\rho_{H_2O} = 1000\ kg/m^3$)



- Assume the pressure is uniform in the tank, thus we can determine the pressure at the gage port.

- Analysis

Starting with P_g (gas pressure) and moving along the tube by adding (as we go down) or subtracting (as we go up), the $\rho g h$ term(s) until we reach point 2, therefore:

$$P_g - \rho g h = P_2$$

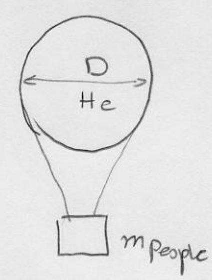
Since tube at point 2 is open to atomsphere, $\underline{P_2 = P_{atm}}$

$$\underbrace{P_g - P_2}_{P_{gage}} = \rho g h$$

(what we read on the pressure gage)

$$80 (kPa) = \rho g h \quad \begin{cases} H_2O \rightarrow h_{H_2O} = \underline{8.155 (m} \\ Hg \rightarrow h_{Hg} = 0.599 (m \end{cases}$$

Problem 2 & 3



$D = 10\,(m)$

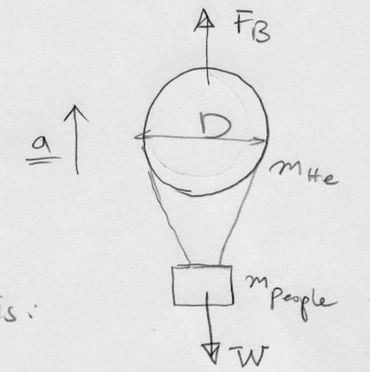
$\rho_{He} = \frac{1}{7}\rho_{air}$, $\rho_{air} = 1.16\,(kg/m^3)$

$m_{people} = 140\,(kg)$

– neglect the weight of the ropes and the cage.

Find : a ? (acceleration)

Analysis



(Free body diagram)

Starting with free body diagram.

We also know that the buoyancy force is:

$$F_B = \rho_{air} \cdot g \cdot V_{ballon}.$$

$$W = m_{total} \cdot g$$

$$\sum F = m_{Total} \cdot a \quad (1)$$

$$m_{total} = m_{people} + m_{He}$$

$$m_{He} = \rho_{He} \cdot V_{ballon}$$

$$V_{ballon} = \frac{4}{3}\pi R^3_{ballon}$$

$$V_{ballon} = \frac{4}{3}\pi (5)^3 = 523.59\,(m^3)$$

$$m_{He} = \frac{1}{7} \times 1.16\,(kg/m^3) \times 523.59\,(m^3) = 86.77\,(kg)$$

$$m_{Total} = m_{He} + m_{People} = 86.77 + 140 = 226.77\,(kg)$$

equation (1) becomes:

$$F_B - W = m_{total} \cdot a \quad (2)$$

$$\rho_{air} \cdot g \cdot V_{ballon} - m_{total} \cdot g = m_{total} \cdot a$$

$$a = \frac{\rho_{air} V_{ballon} - m_{total}}{m_{total}} \cdot g \quad (3)$$

Subtituting values in (3):

$$a = \frac{1.16\,(kg/m^3) \times 523.59\,(m^3) - 226.77\,(kg)}{226.77\,(kg)} \times 9.81\,(m/s^2)$$

$$\boxed{a = 16.46\,(m/s^2)}$$

The max amount of load that the ballon can carry can be calculated from:

$$\Sigma F = 0$$

From equation (2) $\rightarrow$ $F_B = W$

$$\rho_{air} \cdot \cancel{g} \cdot V_{ballon} = m_{max} \cdot \cancel{g}$$

$$m_{max} = \rho_{air} \cdot V_{ballon}$$

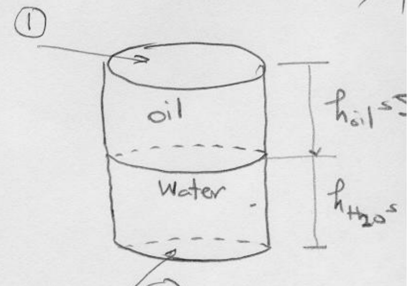
$$= 1.16\,(kg/m^3) \times 523.59\,(m^3)$$

$$m_{max} = 607.304$$ (including the hellium gas in the ballon)

$$m_{Max.\ Load} = m_{max} - m_{He}$$

$$m_{max} = 607.304 - 86.77 = 520.6\,(kg)$$

Problem - 4



$$\rho_{oil} / \rho_{H_2O} = 0.85$$

Find: $P_2 - P_1 = ?$

Analysis:

Starting with point 1 and adding terms $\rho g h$ as we go down.

$$P_1 + \rho_{oil} \cdot g \cdot h_{oil} + \rho_{H_2O} \cdot g \cdot h_{H_2O} = P_2$$

$$P_2 - P_1 = \rho_{oil} \cdot g \cdot h_{oil} + \rho_{H_2O} \cdot g \cdot h_{H_2O}$$

$$P_2 - P_1 = \left(\frac{\rho_{oil}}{\rho_{H_2O}} \cdot h_{oil} + h_{H_2O} \right) \cdot \rho \cdot g$$

Subtituting values:

$\rho_{H_2O} = 1000 \; (kg/m^3)$

$$P_2 - P_1 = 1000 \; (kg/m^3) \left[0.85 \, (-) \times 5 \, (m) + 5 \, (m) \right] \times 9.81 \, (m/$$

$$P_2 - P_1 = 90742.5 \; (Pa) = \underline{90.742 \; (kPa)}$$