Introduction to Thermodynamics and Heat Transfer (ECE 309)
Suggested Problems for Chapter 5

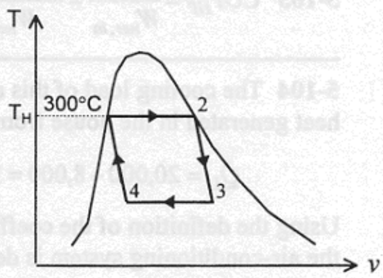
1. Consider a Carnot heat engine cycle executed in a steady-flow system using steam as the working fluid. The cycle has a thermal efficiency of 30 percent, and steam changes from saturated liquid to saturated vapor at 300°C during the heat addition process. If the mass flow rate of the steam is 5 kg/s, determine the net power output of this engine, in kW.

The enthalpy of vaporization h_{fg} at a given T or P represents the amount of heat transfer as 1 kg of a substance is converted from saturated liquid to saturated vapor at that T or P. Therefore, the rate of heat transfer to the steam during heat addition process is

$$\dot{Q}_H = \dot{m} h_{fg\,@\,300^\circ C} = (5\ \text{kg/s})(1404.9\ \text{kJ/kg}) = 7025\ \text{kJ/s}$$

Then the power output of this heat engine becomes

$$\dot{W}_{net,out} = \eta_{th}\dot{Q}_H = (0.30)(7025\ \text{kW}) = \mathbf{2107.5\ kW}$$



2. Consider a Carnot heat engine cycle executed in a closed system using 0.0103 kg of steam as the working fluid. It is known that the maximum absolute temperature in the cycle is twice the minimum absolute temperature, and the net work output of the cycle is 25 kJ. If the steam changes from saturated vapor to saturated liquid during the heat rejection process, determine the temperature of the steam during the heat rejection process, in °C.

The thermal efficiency of the cycle is

$$\eta_{th} = 1 - \frac{T_L}{T_H} = 1 - \frac{1}{2} = 0.5$$

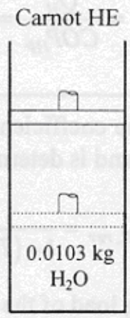
Also,

$$\eta_{th} = \frac{W}{Q_H} \longrightarrow Q_H = \frac{W}{\eta_{th}} = \frac{25\ \text{kJ}}{0.5} = 50\ \text{kJ}$$

$$Q_L = Q_H - W = 50 - 25 = 25\ \text{kJ}$$

and

$$q_L = \frac{Q_L}{m} = \frac{25\ \text{kJ}}{0.0103\ \text{kg}} = 2427\ \text{kJ/kg} = h_{fg\,@\,T_L}$$



since the enthalpy of vaporization h_{fg} at a given T or P represents the amount of heat transfer as 1 kg of a substance is converted from saturated liquid to saturated vapor at that T or P. Therefore, T_L is the temperature which corresponds to the h_{fg} value of 2427 kJ/kg, and is determined from the steam tables to be

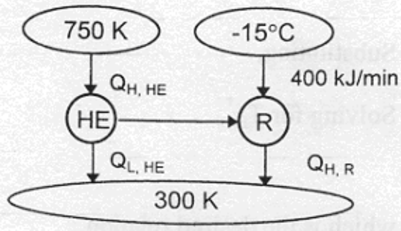
T_L = **31.5°C**

3. A Carnot heat engine receives heat at 750 K and rejects the waste heat to the environment at 300 K. The entire work output of the heat engine is used to drive a Carnot refrigerator that removes heat from the cooled space at -15°C at a rate of 400 kJ /min and rejects it to the same environment at 300 K. Determine (a) the rate of heat supplied to the heat engine and (b) the total rate of heat rejection to the environment.

(a) The coefficient of performance of the Carnot refrigerator is

$$COP_{R,C} = \frac{1}{(T_H / T_L) - 1} = \frac{1}{(300\text{ K})/(258\text{ K}) - 1} = 6.14$$

Then power input to the refrigerator becomes



$$\dot{W}_{net,in} = \frac{\dot{Q}_L}{COP_{R,C}} = \frac{400\text{ kJ/min}}{6.14} = 65.1\text{ kJ/min}$$

which is equal to the power output of the heat engine, $\dot{W}_{net,out}$.

The thermal efficiency of the Carnot heat engine is determined from

$$\eta_{th,C} = 1 - \frac{T_L}{T_H} = 1 - \frac{300\text{ K}}{750\text{ K}} = 0.60$$

Then the rate of heat input to this heat engine is determined from the definition of thermal efficiency to be

$$\dot{Q}_{H,HE} = \frac{\dot{W}_{net,out}}{\eta_{th,HE}} = \frac{65.1\text{ kJ/min}}{0.60} = \mathbf{108.5\text{ kJ/min}}$$

(b) The total rate of heat rejection to the ambient air is the sum of the heat rejected by the heat engine ($\dot{Q}_{L,HE}$) and the heat discarded by the refrigerator ($\dot{Q}_{H,R}$),

$$\dot{Q}_{L,HE} = \dot{Q}_{H,HE} - \dot{W}_{net,out} = 108.5 - 65.1 = 43.4\text{ kJ/min}$$

$$\dot{Q}_{H,R} = \dot{Q}_{L,R} - \dot{W}_{net,in} = 400 - 65.1 = 465.1\text{ kJ/min}$$

and

$$\dot{Q}_{Ambient} = \dot{Q}_{L,HE} + \dot{Q}_{H,R} = 43.4 + 465.1 = \mathbf{508.5\text{ kJ/min}}$$

4. A heat engine operates between two reservoirs at 800 and 20°C. One-half of the work output of the heat engine is used to drive a Carnot heat pump that removes heat from the cold surroundings at 2°C and transfers it to a house maintained at 22°C. If the house is losing heat at a rate of 95,000 kJ/h, determine the minimum rate of heat supply to the heat engine required to keep the house at 22°C.

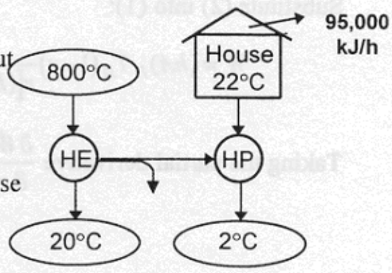
The coefficient of performance of the Carnot heat pump is

$$COP_{HP,C} = \frac{1}{1 - (T_L / T_H)} = \frac{1}{1 - (2 + 273\text{ K})/(22 + 273\text{ K})} = 14.75$$

Then power input to the heat pump, which is supplying heat to the house at a rate of 80,000 kJ/h, becomes

$$\dot{W}_{net,in} = \frac{\dot{Q}_H}{COP_{HP,C}} = \frac{95{,}000\text{ kJ/h}}{14.75} = 6441\text{ kJ/h}$$

which is half the power produced by the heat engine. Thus the power output of the heat engine is



$$\dot{W}_{net,out} = 2\dot{W}_{net,in} = (2)(6441\text{ kJ/h}) = 12{,}882\text{ kJ/h}$$

To minimize the rate of heat supply, we must use a Carnot heat engine whose thermal efficiency is determined from

$$\eta_{th,C} = 1 - \frac{T_L}{T_H} = 1 - \frac{293\text{ K}}{1073\text{ K}} = 0.727$$

Then the rate of heat supply to this heat engine is determined from the definition of thermal efficiency to be

$$\dot{Q}_{H,HE} = \frac{\dot{W}_{net,out}}{\eta_{th,HE}} = \frac{12{,}882\text{ kJ/h}}{0.727} = \mathbf{17{,}719\text{ kJ/h}}$$