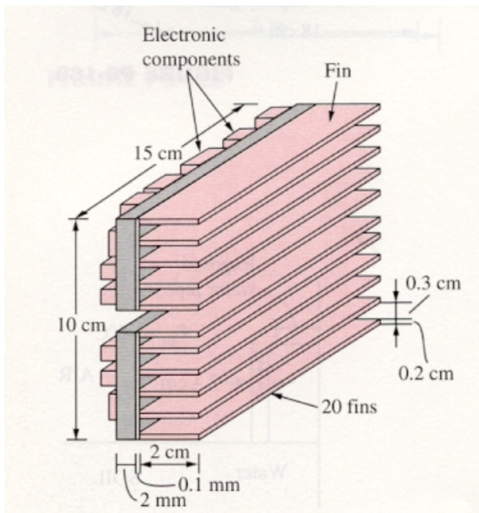
Introduction to Thermodynamics and Heat Transfer (ECE 309)
Suggested Problems for Chapter 8,9

1. A 0.2-cm-thick, 10-cm-high, and 15-cm-long circuit board houses electronic components on one side that dissipate a total of 15 W of heal uniformly. The board is impregnated with conducting metal fillings, and has an effective thermal conductivity of 12 W/(m.°C). All the heat generated in the components is conducted across the circuit board, and is dissipated from the back side of the board to a medium at 37 °C, with a heat transfer coefficient of 45 W /(m2.°C). (a) Determine the surface temperatures on the two sides of the circuit board. (b) Now a 0.l-cm-thick, 10-cm-high, and 15-cm-long aluminum plate [k = 237 W/(m .°C)] with 20 0.2-cm-thick, 2-cm-long, and 15-cm-wide aluminum fins of rectangular profile are attached to the back side of the circuit board with a 0.015-cm-thick epoxy adhesive [k = 1.8 W/(m.°C)]. Determine the new temperatures on the two sides of the circuit board.



The thermal resistance of the board and the convection resistance on the back side of the board are

$$R_{board} = \frac{L}{kA} = \frac{0.002\ \text{m}}{(12\ \text{W/m.°C})(0.1\ \text{m})(0.15\ \text{m})} = 0.011\ \text{°C/W}$$

$$R_{conv} = \frac{1}{hA} = \frac{1}{(45\ \text{W/m.°C})(0.1\ \text{m})(0.15\ \text{m})} = 1.481\ \text{°C/W}$$

$$R_{total} = R_{board} + R_{conv} = 0.011 + 1.481 = 1.492\ \text{°C/W}$$

Then surface temperatures on the two sides of the circuit board becomes

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{total} = 37\text{°C} + (15\ \text{W})(1.492\ \text{°C/W}) = \mathbf{59.4°C}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{board}} \longrightarrow T_2 = T_1 - \dot{Q}R_{board} = 59.4\text{°C} - (15\ \text{W})(0.011\ \text{°C/W}) = \mathbf{59.2°C}$$

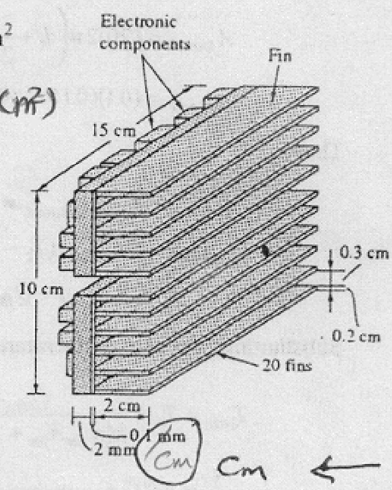
(b) The efficiency of the fins is determined from Fig. 8-59 to be 0.205

$$\xi = \left(L + \frac{t}{2}\right)\sqrt{\frac{h}{kt}} = \left(0.02\ \text{m} + \frac{0.002}{2}\ \text{m}\right)\sqrt{\frac{45\ \text{W/m}^2.\text{°C}}{(237\ \text{W/m.°C})(0.002\ \text{m})}} = 0.205 \longrightarrow \eta_{fin} = 0.96$$

The finned and unfinned surface areas are

$$A_{finned} = (20)2w\left(L + \frac{t}{2}\right) = (20)2(0.15)\left(0.02 + \frac{0.002}{2}\right) = 0.126\ \text{m}^2$$

$$A_{unfinned} = (0.1)(0.15) - (0.002)(0.15) \times 20 = 0.009\ (\text{m}^2)$$



Cm ←

Then,

$$\dot{Q}_{finned} = \eta_{fin}\dot{Q}_{fin,\max} = \eta_{fin} h A_{fin}(T_{base} - T_\infty)$$

$$\dot{Q}_{unfinned} = hA_{unfined}(T_{base} - T_\infty)$$

$$\dot{Q}_{total} = \dot{Q}_{unfinned} + \dot{Q}_{finned} = h(T_{base} - T_\infty)(\eta_{fin}A_{fin} + A_{unfined})$$

Substituting, the base temperature of the finned surfaceis determine to be

$$T_{base} = T_\infty + \frac{\dot{Q}_{total}}{h(\eta_{fin}A_{fin} + A_{unfined})}$$

$$= 37\text{°C} + \frac{15\ \text{W}}{(45\ \text{W/m}^2.\text{°C})[(0.96)(0.126\ \text{m}^2) + (.009\ \text{m}^2)]}$$ ←

$$= \mathbf{39.5°C}$$ ✓

Then the temperatures on both sides of the board are determined using the thermal resistance network to be

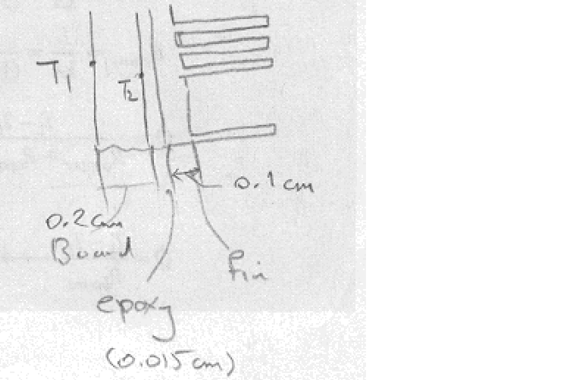
$$R_{aluminum} = \frac{L}{kA} = \frac{0.001\ \text{m}}{(237\ \text{W/m.°C})(0.1\ \text{m})(0.15\ \text{m})} = 0.00028\ \text{°C/W}$$

$$R_{epoxy} = \frac{L}{kA} = \frac{0.00015\ \text{m}}{(1.8\ \text{W/m.°C})(0.1\ \text{m})(0.15\ \text{m})} = 0.00555\ \text{°C/W}$$

$$\dot{Q} = \frac{T_1 - T_{base}}{R_{\text{aluminum}} + R_{\text{epoxy}} + R_{\text{board}}} = \frac{(T_1 - 39.5)\text{°C}}{(0.00028 + 0.00555 + 0.011)\ \text{°C/W}}$$

$$\longrightarrow T_1 = 39.5\text{°C} + (15\ \text{W})(0.0168\ \text{°C/W}) = \mathbf{39.8°C}$$

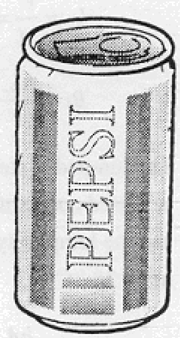
$$\dot{Q} = \frac{T_1 - T_2}{R_{board}} \longrightarrow T_2 = T_1 - \dot{Q}R_{board} = 39.8\text{°C} - (15\ \text{W})(0.011\ \text{°C/W}) = \mathbf{39.6°C}$$



2. During a picnic on a hot summer day all the cold drinks disappeared quickly, and the only available drinks were those at the ambient temperature of 25°C. In an effort to cool a 335-ml drink in a can, which is 12.5 cm high and has a diameter of 6.5 cm, a person grabs the can and starts shaking it in the iced water of the chest at 0°C. The temperature of the drink can be assumed to be uniform at all times, and the heat transfer coefficient between the iced water and the aluminum can is 170 W(m^2.°C). Using the properties of water for the drink, estimate how long it will take for the canned drink to cool to 8°C.

Since the temperature of the drink can be assumed to be uniform at all times, the lumped system analysis is applicable. Then,

$$L_c = \frac{V}{A} = \frac{\pi r_o^2 L}{2\pi r_o L + 2\pi r_o^2} = \frac{\pi(0.0325\text{ m})^2(0.125\text{ m})}{2\pi(0.0325\text{ m})(0.125\text{ m}) + 2\pi(0.0325\text{ m})^2} = 0.0129\text{ m}$$



and

$$b = \frac{hA}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{170\text{ W/m}^2.°\text{C}}{(1000\text{ kg/m}^3)(4180\text{ J/kg.°C})(0.0129\text{ m})} = 0.00315\text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{8-0}{25-0} = e^{-(0.00315\text{ s}^{-1})t} \longrightarrow t = \mathbf{362\text{ s}}$$

Therefore, it will take about 6 minutes to cool the canned drink to 8°C.