1. A 0.2-cm-thick, 10-cm-high, and 15-cm-long circuit board houses electronic components on one side that dissipate a total of 15 W of heat uniformly. The board is impregnated with conducting metal fillings, and has an effective thermal conductivity of 12 W/(m°C). All the heat generated in the components is conducted across the circuit board, and is dissipated from the back side of the board to a medium at 37 °C, with a heat transfer coefficient of 45 W/(m²°C). (a) Determine the surface temperatures on the two sides of the circuit board. (b) Now a 0.1-cm-thick, 10-cm-high, and 15-cm-long aluminum plate [k = 237 W/(m°C)] with 20 0.2-cm-thick, 2-cm-long, and 15-cm-wide aluminum fins of rectangular profile are attached to the back side of the circuit board with a 0.015-cm-thick epoxy adhesive [k = 1.8 W/(m°C)]. Determine the new temperatures on the two sides of the circuit board.
The thermal resistance of the board and the convection resistance on the back side of the board are

\[ R_{\text{board}} = \frac{L}{kA} = \frac{0.002 \text{ m}}{(12 \text{ W/m}^\circ \text{C})(0.1 \text{ m})(0.15 \text{ m})} = 0.011 \circ \text{C/W} \]

\[ R_{\text{conv}} = \frac{1}{hA} = \frac{1}{(45 \text{ W/m}^\circ \text{C})(0.1 \text{ m})(0.15 \text{ m})} = 1.481 \circ \text{C/W} \]

\[ R_{\text{total}} = R_{\text{board}} + R_{\text{conv}} = 0.011 + 1.481 = 1.492 \circ \text{C/W} \]

Then surface temperatures on the two sides of the circuit board becomes

\[ \frac{Q}{R_{\text{total}}} = \frac{T_i - T_\infty}{T_i - T_\infty + \frac{QR_{\text{total}}}{(15 \text{ W})(1.492 \circ \text{C/W})} = 59.4 \circ \text{C} \]

\[ \frac{Q}{R_{\text{board}}} = \frac{T_i - T_\infty}{T_i - T_\infty + \frac{QR_{\text{board}}}{(15 \text{ W})(0.011 \circ \text{C/W})} = 59.2 \circ \text{C} \]

(b) The efficiency of the fins is determined from Fig. 8-59 to be

\[ \eta = 0.205 \]

The finned and unfinned surface areas are

\[ A_{\text{finned}} = (20)2w \left( L + \frac{L}{2} \right) = (20)(2)(0.15)(0.02 + \frac{0.002}{2}) = 0.126 \text{ m}^2 \]

\[ A_{\text{unfinned}} = (0.1)(0.15) - 0.002)(0.15)(2.0 \times 2.0 = 0.009 \text{ m}^2 \]

Substituting the base temperature of the finned surface is determined to be

\[ T_{\text{base}} = T_\infty + \frac{Q_{\text{total}}}{h(\eta_{\text{fin}}A_{\text{fin}} + A_{\text{unfinned}})} = 37 \circ \text{C} + \frac{15 \text{ W}}{(45 \text{ W/m}^\circ \text{C})(0.96)(0.126 \text{ m}^2) + (0.009 \text{ m}^2)} \]

\[ = 39.5 \circ \text{C} \]

Then the temperatures on both sides of the board are determined using the thermal resistance network to be

\[ R_{\text{aluminum}} = \frac{L}{kA} = \frac{0.001 \text{ m}}{(237 \text{ W/m}^\circ \text{C})(0.1 \text{ m})(0.15 \text{ m})} = 0.00028 \circ \text{C/W} \]

\[ R_{\text{epoxy}} = \frac{L}{kA} = \frac{0.00015 \text{ m}}{(18 \text{ W/m}^\circ \text{C})(0.1 \text{ m})(0.15 \text{ m})} = 0.00555 \circ \text{C/W} \]

\[ Q = \frac{T_i - T_{\text{base}}}{R_{\text{aluminum}} + R_{\text{epoxy}} + R_{\text{board}}} = \frac{39.5 \circ \text{C}}{(0.00028 + 0.00555 + 0.00011) \circ \text{C/W}} \]

\[ = \frac{39.5 \circ \text{C}}{0.00695 \circ \text{C/W}} \rightarrow T_i = 39.5 \circ \text{C} + (15 \text{ W})(0.0168 \circ \text{C/W}) = 39.8 \circ \text{C} \]

\[ Q = \frac{T_i - T_2}{R_{\text{board}}} \rightarrow T_2 = T_i - \frac{QR_{\text{board}}}{(15 \text{ W})(0.011 \circ \text{C/W})} = 39.8 \circ \text{C} - (15 \text{ W})(0.011 \circ \text{C/W}) = 39.6 \circ \text{C} \]
2. During a picnic on a hot summer day all the cold drinks disappeared quickly, and the only available drinks were those at the ambient temperature of 25°C. In an effort to cool a 335-ml drink in a can, which is 12.5 cm high and has a diameter of 6.5 cm, a person grabs the can and starts shaking it in the iced water of the chest at 0°C. The temperature of the drink can be assumed to be uniform at all times, and the heat transfer coefficient between the iced water and the aluminum can is 170 W(m²°C). Using the properties of water for the drink, estimate how long it will take for the canned drink to cool to 8°C.

Since the temperature of the drink can be assumed to be uniform at all times, the lumped system analysis is applicable. Then,

\[ L_c = \frac{V}{A} = \frac{\pi d^2 L}{2\pi d L + 2\pi d^2} = \frac{\pi (0.0325 \text{ m})^2 (0.125 \text{ m})}{2\pi (0.0325 \text{ m})(0.125 \text{ m}) + 2\pi (0.0325 \text{ m})^2} = 0.0129 \text{ m} \]

and

\[ b = \frac{hA}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{170 \text{ W/m²°C}}{(1000 \text{ kg/m}^3)(4180 \text{ J/kg°C})(0.0129 \text{ m})} = 0.00315 \text{ s}^{-1} \]

\[ \frac{T(t) - T_c}{T_f - T_c} = e^{-bt} \rightarrow \frac{8 - 0}{25 - 0} = e^{-\left(0.00315 \text{ s}^{-1}\right)t} \rightarrow t = 362 \text{ s} \]

Therefore, it will take about 6 minutes to cool the canned drink to 8°C.