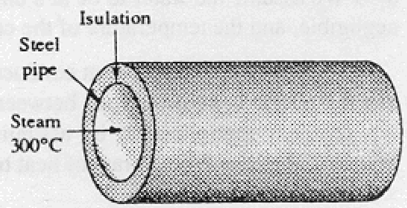
Introduction to Thermodynamics and Heat Transfer (ECE 309)
Suggested Problems for Chapter 8

1. Steam at 300 °C is flowing through a steel pipe [k = 15.1 W/(m.°C)] whose inner and outer diameters are 8 and 8.8 cm, respectively, in an environment at 15 °C. The pipe is insulated with 3-cm-thick fiberglass insulation [k = 0.035 W/(m.°C)]. If the heat transfer coefficients on the inside and the outside of the pipe are 150 and 25W/(m2.°C), respectively, determine the rate of heat loss from the steam per meter length of the pipe. What is the error involved in neglecting the thermal resistance of the steel pipe in calculations?

The inner and the outer surface areas of the insulated pipe are

$$A_i = \pi D_i L = \pi(0.08\text{ m})(1\text{ m}) = 0.251\text{ m}^2$$
$$A_o = \pi D_o L = \pi(0.148\text{ m})(1\text{ m}) = 0.465\text{ m}^2$$



The individual resistances are

$$R_{conv,i} = \frac{1}{h_i A_i} = \frac{1}{(150\text{ W/m}^2.°\text{C})(0.251\text{ m}^2)} = 0.027\text{ °C/W}$$

$$R_1 = R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(4.4/4)}{2\pi(15.1\text{ W/m.°C})(1\text{ m})} = 0.001\text{ °C/W}$$

$$R_2 = R_{insulation} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(7.4/4.4)}{2\pi(0.035\text{ W/m.°C})(1\text{ m})} = 2.364\text{ °C/W}$$

$$R_{conv,o} = \frac{1}{h_o A_o} = \frac{1}{(25\text{ W/m}^2.°\text{C})(0.465\text{ m}^2)} = 0.086\text{ °C/W}$$

$$R_{total} = R_{conv,i} + R_1 + R_2 + R_{conv,o} = 0.027 + 0.001 + 2.364 + 0.086 = 2.478\text{ °C/W}$$

Then the steady rate of heat loss from the steam per m length of the pipe becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(300-15)°\text{C}}{2.478\text{ °C/W}} = \mathbf{115.01\text{ W}}$$

If the thermal resistance of the steel pipe is neglected, the new value of total thermal resistance will be

$$R_{total} = R_{conv,i} + R_2 + R_{conv,o} = 0.027 + 2.364 + 0.086 = 2.477\text{ °C/W}$$

Then the percentage error involved in calculations becomes

$$error\% = \frac{(2.478 - 2.477)°\text{C}}{2.478\text{ °C}} \times 100 = \mathbf{0.04\%}$$

which is insignificant.

2. The plumbing system of a house involves a 0.5-m section of a plastic pipe [k = 0.16 W/(m.°C)] of inner diameter 2 cm and outer diameter 2.4 cm, exposed to the ambient air. During a cold and windy night, the ambient air temperature remains at about –5 °C for a period of 14 h. The combined convection and radiation heat transfer coefficient on the outer surface of the pipe is estimated to be 40 W/(m^2.°C), and the heat of fusion of water is 333.7 kJ /kg. Assuming the pipe to contain stationary water initially at 0 °C, determine if the water in that section of the pipe will completely freeze that night.

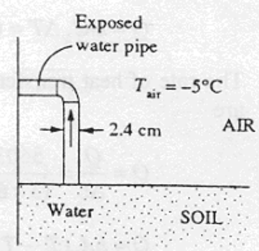
We take the conservative approach and assume the inner surface of the pipe to be at 0°C. The thermal resistances involved and the rate of heat transfer are

$$R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(1.2/1)}{2\pi(0.16\ \text{W/m.°C})(0.5\ \text{m})} = 0.3627\ \text{°C/W}$$

$$R_{conv,o} = \frac{1}{h_o A} = \frac{1}{(40\ \text{W/m}^2\text{.°C})[\pi(0.024\ \text{m})(0.5\ \text{m})]} = 0.6631\ \text{°C/W}$$

$$R_{total} = R_{pipe} + R_{conv,o} = 0.3627 + 0.6631 = 1.0258\ \text{°C/W}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[0-(-5)]\text{°C}}{1.0258\ \text{°C/W}} = 4.87\ \text{W}$$



The total amount of heat lost by the water during a 14-h period that night is

$$Q = \dot{Q}\Delta t = (4.87\ \text{J/s})(14 \times 3600\ \text{s}) = 245.65\ \text{kJ}$$

The amount of heat required to freeze the water in the pipe completely is

$$m = \rho V = \rho\pi r^2 L = (1000\ \text{kg/m}^3)\pi(0.01\ \text{m})^2(0.5\ \text{m}) = 0.157\ \text{kg}$$

$$Q = mh_{fg} = (0.157\ \text{kg})(333.7\ \text{kJ/kg}) = 52.4\ \text{kJ}$$

The water in the pipe will **freeze completely** that night since the amount heat loss is greater than the amount it takes to freeze the the water completely(245.65 > 52.4) .