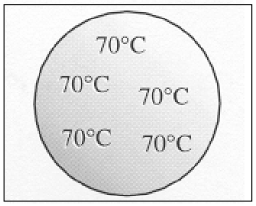
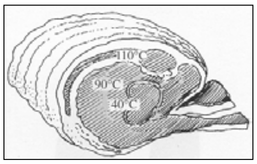
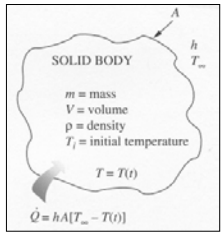
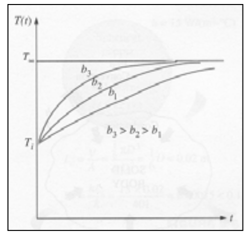
Lumped System Analysis





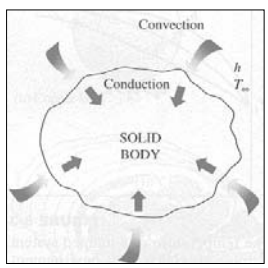




$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$

$$b = \frac{hA}{\rho V C_P}$$

Criteria for Lumped System Analysis



$$\text{Biot Number} = \text{Bi} = \frac{\text{Convection at the surface of the body}}{\text{conduction within the body}} = \frac{h\Delta T}{k\Delta T / L}$$

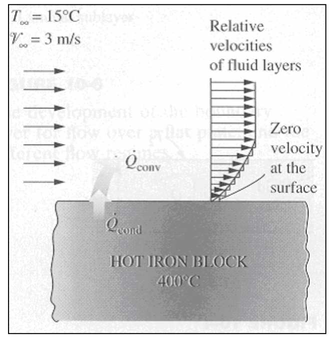
$$L = \text{Characteristic Length} = V / A$$

$$Bi = \frac{hV}{kA} \leq 0.1$$

Convection Heat Transfer

Convection heat transfer strongly depends on the:

- fluid properties,
- dynamic viscosity μ,
- thermal conductivity k,
- density ρ,
- specific heat Cp,
- fluid velocity V,
- geometry of the solid surface,
- type of fluid flow



$$\dot{q}_{Conv} = \dot{q}_{Cond} = -k_{fluid} \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

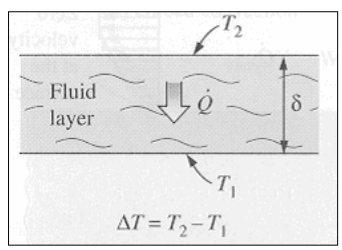
$$h = \frac{-k_{fluid}(\partial T / \partial y)_{y=0}}{T_s - T_\infty}$$

Nusselt Number, Nu

$$\dot{q}_{conv} = h\,\Delta T$$

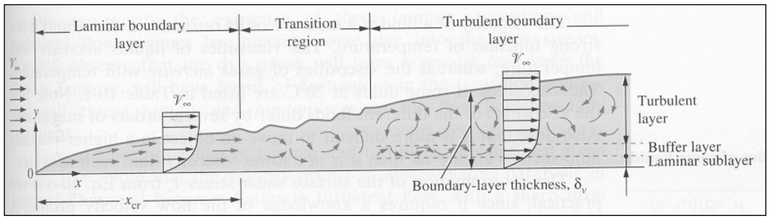
$$\dot{q}_{cond} = k\frac{\Delta T}{\delta}$$

$$\frac{\dot{q}_{conv}}{\dot{q}_{cond}} = \frac{h\delta}{k} = \text{Nu}$$



- The larger the Nusselt number, the more effective is the convection. A Nusselt number **Nu = 1** for a fluid layer represents heat transfer by ***pure conduction***.

Velocity Boundary Layer

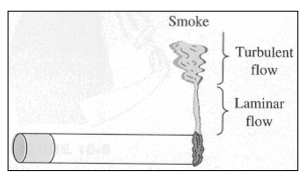


- The region of the flow above the plate bounded by δ_v in which the effects of the viscous shearing forces caused by fluid viscosity are felt is called the ***velocity boundary layer.***

- The *thickness* of the boundary layer, δ_v, is arbitrarily defined as the distance from the surface at which $V = 0.99\ V_\infty$.

Laminar & Turbulent Flows

- ***Laminar flow*** is characterized by smooth streamlines and highly ordered motion,
- ***Turbulent Flow*** is characterized by velocity fluctuations and highly disordered motion.



$$\text{Re} = \frac{\text{inertia forces}}{\text{viscouse fources}} = \frac{V_\infty \delta}{\nu}$$

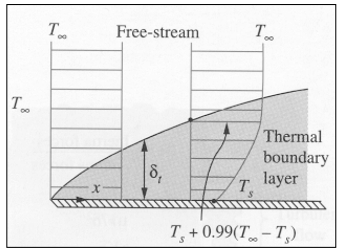
$$\begin{cases} V_\infty & \text{Free-stream velocity [m/s]} \\ \delta & \text{Characteristic length, [m]} \\ \nu & \text{Kinematic viscosity } (\mu/\rho), [\text{m}^2/s] \end{cases}$$

- ***Critical Reynolds number is*** the Reynolds number at which the flow becomes turbulent.

$$\text{Re}_{\text{critical, flat plate}} \approx 5\times 10^5$$

Thermal Boundary Layer

- A ***thermal boundary layer*** develops when a fluid at a specified temperature flows over a surface which is at a different temperature.
- The ***thickness*** of the thermal boundary layer δt at any location is defined as the distance from the surface at which the temperature difference (T-Ts) equals 0.99(T∞-Ts).
- The thickness of the thermal boundary layer ***increases*** in the flow direction.
- The ***relative thickness*** of the velocity and the thermal boundary layers is described by Prandtl Number.



$$\Pr = \frac{\nu}{\alpha} = \frac{\mu C_P}{k}$$

$$\begin{cases} \Pr \approx 1 & \text{Most Gases,} \\ \Pr << 1 & \text{Liquid Metals,} \\ \Pr >> 1 & \text{Oils.} \end{cases}$$

Flow Over Flat Plates

- Laminar:

$$Nu_x = \frac{h_x x}{k} = 0.332 \mathrm{Re}_x^{1/2} \Pr^{1/3} \qquad (\Pr \geq 0.6)$$

$$Nu = \frac{hL}{k} = 0.664 \mathrm{Re}_L^{1/2} \Pr^{1/3}$$

- Turbulent:

$$Nu_x = \frac{h_x x}{k} = 0.0296 \mathrm{Re}_x^{4/5} \Pr^{1/3} \qquad \begin{pmatrix} 0.6 \leq \Pr \leq 60 \\ 5\times 10^5 \leq \mathrm{Re}_x \leq 10^7 \end{pmatrix}$$

$$Nu = \frac{hL}{k} = 0.037 \mathrm{Re}_L^{4/5} \Pr^{1/3} \qquad \begin{pmatrix} 0.6 \leq \Pr \leq 60 \\ 5\times 10^5 \leq \mathrm{Re}_L \leq 10^7 \end{pmatrix}$$

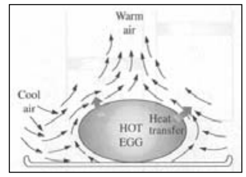
- Combined:

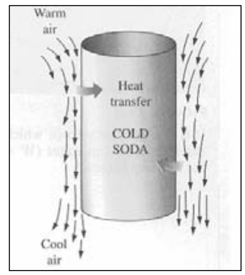
$$h = \frac{1}{L}\left(\int_0^{x_{cr}} h_{x,laminar} dx + \int_{x_{cr}}^{L} h_{x,turbulent} dx \right)$$

- All properties are evaluated at film temperature as: $T_f = \dfrac{T_s + T_\infty}{2}$

Natural Convection, Physical Mechanism

- The magnitude of the natural convection heat transfer between a surface and a fluid is directly related to the ***mass flow rate*** of the fluid.
- The higher the mass flow rate, the higher is the heat transfer rate.
- The mass flow is established by the dynamic balance of ***buoyancy*** and ***friction***.



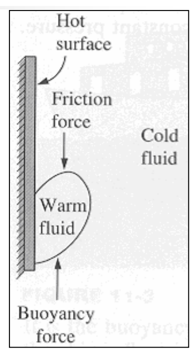


Grashof Number, Gr

- Grashof Number, Gr, is a measure of the relative magnitudes of the buoyancy force and the opposing friction force acting on the fluid.

$$\text{Gr} = \frac{\text{buoyancy forces}}{\text{viscous forces}} = \frac{g\,\Delta\rho V}{\rho\nu^2} = \frac{g\,\beta\Delta T\,V}{\rho\nu} = \frac{g\beta(T_s - T_\infty)\delta^3}{\nu^2}$$

$$\begin{cases} g & \text{Gravitational acceleration [m/s}^2\text{]} \\ \beta & \text{Coeff. of volume expansion, } 1/T, \text{[1/K]} \\ \Delta T & T_s - T_\infty, \text{[°C]} \\ \delta & \text{Characteristic length, [m]} \\ \nu & \text{kinematic viscosity, } \mu/\rho, \text{[m}^2\text{/s]} \end{cases}$$



- The Grashof number provides the main criteria in determining whether the fluid flow is laminar or turbulent in natural convection.

$\text{Gr} \le 10^9$	Laminar
$\text{Gr} > 10^9$	Turbulent

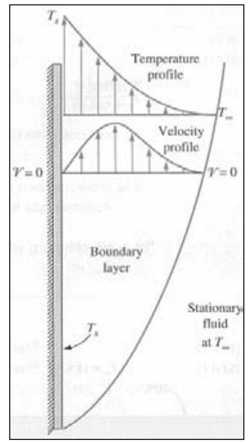
Natural Convection Over Surfaces

$$Nu = \frac{h\delta}{k} = \mathrm{C}(\mathrm{Gr\,Pr})^n = \mathrm{C\,Ra}^n$$

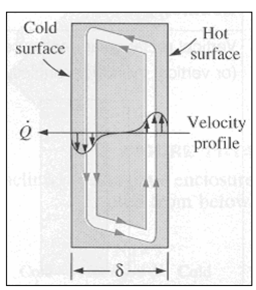
Where ***Rayleigh number***, Ra, is

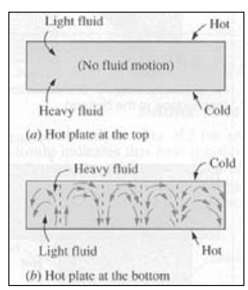
$$Ra = Gr\,\mathrm{Pr} = \frac{g\beta(T_s - T_\infty)\delta^3}{\nu^2}\,\mathrm{Pr}$$

- The values of the constants C and *n* depend on the *geometry* of the surface and the *flow regime,*
- The value of *n* is usually ¼ for laminar flow and 1/3 for turbulent flow.
- The value of the constant C is normally less than 1. ***(Table 11-1, P 587)***



Natural Convection Inside Enclosures





$$Ra = \frac{g\beta\,(T_1 - T_2)\,\delta^3}{\nu^2}\,\mathrm{Pr}$$

$$\begin{cases} \delta & \text{Distance between the hot \& cold plates [m]} \\ T_1, T_2 & \text{Temperatures of hot \& cold surfaces [°C]} \end{cases}$$

$$\dot{Q} = hA_s(T_1 - T_2) = k\,\mathrm{Nu}\,A_s\frac{T_1 - T_2}{\delta}$$

$$A = \begin{cases} HL & \text{rectangular enclosures} \\ \dfrac{\pi L(D_2 - D_1)}{\ln(D_2/D_1)} & \text{concentric cylinders} \\ \pi D_1 D_2 & \text{concentric spheres} \end{cases}$$