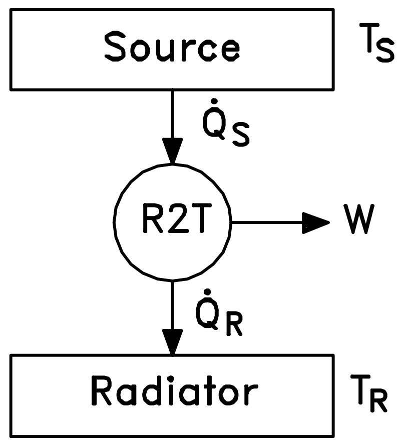
Problem: 7-13

Given: Reversible 2T engine with fixed source temperature and fixed power. Weight of radiator is proportional to its area. Rate of radiation is proportional to $A \cdot T_R^4$

Show: Least weight occurs for $T_{RAD} = 0.75 \cdot T_{source}$



Given $\dot{Q}_R = kAT_R^4$; $k =$ constant

For a reversible 2T engine:

$$\frac{\dot{Q}_S}{\dot{Q}_R} = \frac{T_S}{T_R} \quad \text{since} \quad P_S = 0$$

(this is the maximum performance for a Carnot cycle)

Energy balance:

$$\dot{W} = \dot{Q}_S - \dot{Q}_R = \dot{Q}_S\left(\frac{T_S}{T_R} - 1\right) = kAT_R^4\left(\frac{T_S}{T_R} - 1\right)$$

Then

$$A = \frac{\dot{W}}{k(T_R^3 T_S - T_R^4)}$$

Since weight is proportional to area, for minimum weight we require minimum area. Then, since $\dot{W}$ and T_S are constant:

$$\frac{dA}{dT_R} = \frac{\dot{W}}{k}\frac{(3T_R^2\, T_S - 4T_R^3)}{(T_R^3\, T_S - T_R^4)^2} = 0$$

or

$$3T_R^2\ T_S - 4T_R^3 = 0$$

or

$$\boxed{\frac{T_R}{T_S} = 0.75} \quad \Leftarrow$$

check

$$\left.\frac{d^2A}{dT_R^2}\right|_{T_R=.75T_S} = \frac{1917.83\ \dot{W}}{kT_S^4} > 0$$

therefore it is a minimum.