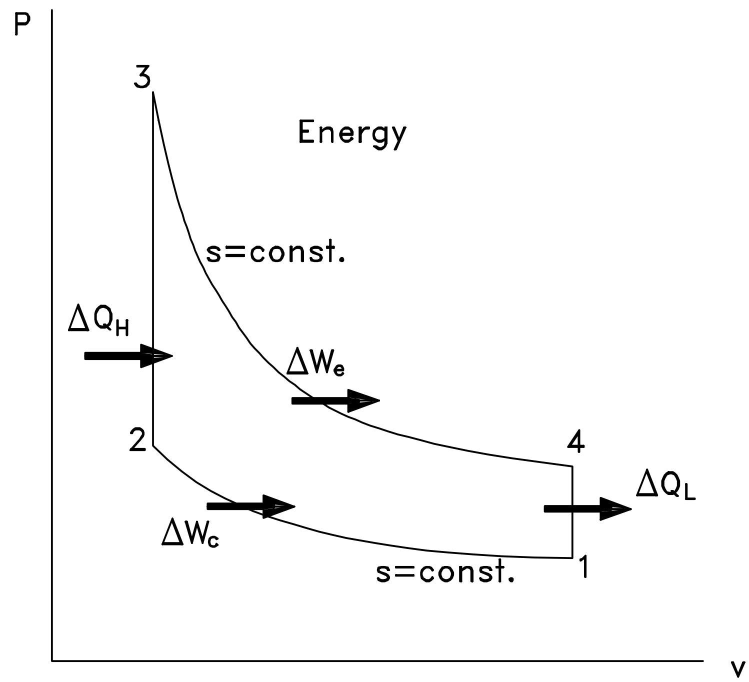
Problem: 6-4

Given: A four-stroke Otto-cycle has a crankshaft speed of 2000 rpm, a compression ratio of 8, and a displacement volume of 1.5 liters. At the start of compression the air is at 293 K, 0.1013 MPa. The peak cycle temperature is equal to the 2000 K source temperature. The compression and expansion processes are each reversible and adiabatic.

Find: i) the energy transfer per unit mass for each process in the cycle

Assume: constant specific heats



The analysis begins with a process representation and evaluation of properties. The given data are shown in the following table:

State	T (K)	P (MPa)	v (m^3/kg)
1	293	0.1013	
2			
3	2000		
4			

An entropy balance for the reversible, adiabatic compression process gives $s_2 - s_1$. The compression ratio gives $v_1/v_2 = 8.0$. The value of T_2 may be found by integrating

$$T\ ds = du + P\ dv$$

for the isentropic process from state (1) to state (2) to obtain

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

Upon multiplying by T_1 and substituting numerical values

$$T_2 = 293 \times 8.0^{(1.4-1)} = 673.1\ K$$

The pressure after compression may be found from the ideal gas equation as

$$P_2 = m_2\ R\ T_2/V_2$$

Since $m_2 = m_1 = (P_1 V_1)/(R\ T_1)$ we may write

$$\begin{aligned} P_2 &= [(P_1V_1)/(R\ T_1)][R\ T_2/V_2] \\ &= P_1(V_1/V_2)(T_2/T_1) \\ &= 0.1013 \times 8.0 \times (673.1/293) \\ &= 1.862\ MPa \end{aligned}$$

The specific volume at state (1) may be computed from the ideal gas equation as

$$\begin{aligned} v_1 &= R\ T_1/P_1 \\ &= \frac{287\ \dfrac{N-m}{kg-K} \times 293\ K}{(0.1013\ MPa \times 10^6\ (N/MPa - m^2))} \\ &= 0.8304\ m^3/kg \end{aligned}$$

The specific volume at state (2) is obtained from v_1 and the compression ratio as

$$v_2 = v_1/(v_1/v_2) = 0.8304/8.0 = 0.1038\ (m^3/kg)$$

The pressure at state (3) may be found from the ideal gas equation as

$$P_3 = m_3\ R\ T_3/V_3$$

Since $m_3 = m_1 = (P_1 V_1)/(R\ T_1)$ and $V_3 = V_2$, we may then write P_3 as

$$\begin{aligned} P_3 &= [(P_1V_1)/(R\ T_1)][R\ T_3/V_2] \\ &= P_1(V_1/V_2)(T_3/T_1) \\ &= 0.1013 \times 8.0 \times (2000/293) \\ &= 5.532\ MPa \end{aligned}$$

State (4) is fixed by $s_4 = s_3$ and $v_4 = v_1$. The value of T_4 can be found by integrating $T\ ds = du + P\ dV$ for the isentropic process from state (3) to state (4) to obtain

$$\frac{T_4}{T_3} = \left(\frac{v_4}{v_3}\right)^{1-k}$$

Since $v_4/v_3 = v_1/v_2 = 8.0$ and $T_3 = T_H = 2000\ K$, we may multiply by T_3 and substitute values to find

$$T_4 = 2000 \times 8.0^{(1-1.4)} = 870.6\ K$$

The pressure at state (4) is given by

$$P_4 = m_4\ R\ T_4/V_4$$

Since $m_4 = m_1$ and $V_4 = V_1 = m_1\ R\ T_1/P_1$ we may write P_4 as

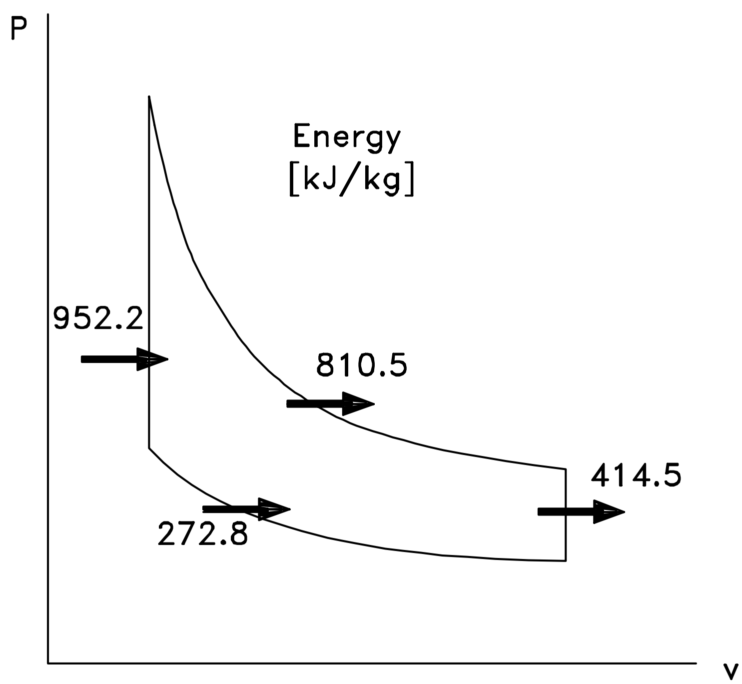
$$P_4 = (T_4/T_1)P_1 = (870.6/293) \times 0.1013 = 0.3010\ MPa$$

The table of property values may now be completed.

State	$\boldsymbol{T}$ $\boldsymbol{(K)}$	$\boldsymbol{P}$ $\boldsymbol{(MPa)}$	$\boldsymbol{v}$ $\boldsymbol{(m^3/kg)}$
1	293	0.1013	0.8304
2	673.1	1.862	0.1038
3	2000	5.532	0.1038
4	870.6	0.3010	0.8304

For constant c_v, internal energy differences may be obtained by integrating $du = c_v dT$. The energy transfer per unit mass may then be found as

$$\begin{aligned} \Delta W_c/m &= c_v(T_2 - T_1) = 0.7176\ (673.1 - 293) &= 272.8\ kJ/kg \\ \Delta Q_H/m &= c_v(T_3 - T_2) = 0.7176\ (2000 - 673.1) &= 952.2\ kJ/kg \\ \Delta W_e/m &= c_v(T_3 - T_4) = 0.7176\ (2000 - 870.6) &= 810.5\ kJ/kg \\ \Delta Q_L/m &= c_v(T_4 - T_1) = 0.7176\ (870.6 - 293) &= 414.5\ kJ/kg \end{aligned}$$


P
Energy
[kJ/kg]
952.2
810.5
414.5
272.8
v