Problem: 6-4

Given: A four-stroke Otto-cycle has a crankshaft speed of 2000 rpm, a compression ratio of 8, and a displacement volume of 1.5 liters. At the start of compression the air is at 293 K, 0.1013 MPa. The peak cycle temperature is equal to the 2000 K source temperature. The compression and expansion processes are each reversible and adiabatic.

Find: i) the energy transfer per unit mass for each process in the cycle

Assume: constant specific heats

The analysis begins with a process representation and evaluation of properties. The given data are shown in the following table:

<table>
<thead>
<tr>
<th>State</th>
<th>$T$ (K)</th>
<th>$P$ (MPa)</th>
<th>$v$ (m$^3$/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>293</td>
<td>0.1013</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
An entropy balance for the reversible, adiabatic compression process gives \( s_2 - s_1 \). The compression ratio gives \( \frac{v_1}{v_2} = 8.0 \). The value of \( T_2 \) may be found by integrating

\[
T \, ds = du + P \, dv
\]

for the isentropic process from state (1) to state (2) to obtain

\[
\frac{T_2}{T_1} = \left( \frac{v_1}{v_2} \right)^{k-1}
\]

Upon multiplying by \( T_1 \) and substituting numerical values

\[
T_2 = 293 \times 8.0^{(1.4-1)} = 673.1 \, K
\]

The pressure after compression may be found from the ideal gas equation as

\[
P_2 = m_2 \frac{R \, T_2}{V_2}
\]

Since \( m_2 = m_1 = \frac{(P_1 V_1)}{(R \, T_1)} \) we may write

\[
P_2 = \left[ \frac{(P_1 V_1)}{(R \, T_1)} \right] [R \, T_2/V_2]
\]

\[
= P_1 (V_1/V_2) (T_2/T_1)
\]

\[
= 0.1013 \times 8.0 \times (673.1/293)
\]

\[
= 1.862 \, MPa
\]

The specific volume at state (1) may be computed from the ideal gas equation as

\[
v_1 = \frac{R \, T_1}{P_1}
\]

\[
= \frac{287 \cdot (N - m)}{kg \cdot K} \times 293 \, K
\]

\[
= \frac{287 \cdot (0.1013 \, MPa \times 10^6 \, (N/MPa - m^2))}{(N/MPa - m^2)}
\]

\[
= 0.8304 \, m^3/kg
\]

The specific volume at state (2) is obtained from \( v_1 \) and the compression ratio as

\[
v_2 = \frac{v_1}{(v_1/v_2)} = \frac{0.8304}{8.0} = 0.1038 \, (m^3/kg)
\]

The pressure at state (3) may be found from the ideal gas equation as

\[
P_3 = m_3 \frac{R \, T_3}{V_3}
\]

Since \( m_3 = m_1 = \frac{(P_1 V_1)}{(R \, T_1)} \) and \( V_3 = V_2 \), we may then write \( P_3 \) as
\[
P_3 = \left(\frac{P_1 V_1}{R T_1}\right) \left(\frac{R T_3}{V_2}\right) = P_1 \left(\frac{V_1}{V_2}\right) \left(\frac{T_3}{T_1}\right) = 0.1013 \times 8.0 \times \left(\frac{2000}{293}\right) = 5.532 \text{ MPa}
\]

State (4) is fixed by \(s_4 = s_3\) and \(v_4 = v_1\). The value of \(T_4\) can be found by integrating \(T \, ds = du + P \, dV\) for the isentropic process from state (3) to state (4) to obtain

\[
\frac{T_4}{T_3} = \left(\frac{v_4}{v_3}\right)^{1-k}
\]

Since \(v_4/v_3 = v_1/v_2 = 8.0\) and \(T_3 = T_H = 2000\ K\), we may multiply by \(T_3\) and substitute values to find

\[
T_4 = 2000 \times 8.0^{(1-1.4)} = 870.6 \text{ K}
\]

The pressure at state (4) is given by

\[
P_4 = m_4 \frac{R T_4}{V_4}
\]

Since \(m_4 = m_1\) and \(V_4 = V_1 = m_1 \frac{R T_1}{P_1}\) we may write \(P_4\) as

\[
P_4 = \left(\frac{T_4}{T_1}\right) P_1 = \left(\frac{870.6}{293}\right) \times 0.1013 = 0.3010 \text{ MPa}
\]

The table of property values may now be completed.

<table>
<thead>
<tr>
<th>State</th>
<th>(T) (K)</th>
<th>(P) (MPa)</th>
<th>(v) (m³/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>293</td>
<td>0.1013</td>
<td>0.8304</td>
</tr>
<tr>
<td>2</td>
<td>673.1</td>
<td>1.862</td>
<td>0.1038</td>
</tr>
<tr>
<td>3</td>
<td>2000</td>
<td>5.532</td>
<td>0.1038</td>
</tr>
<tr>
<td>4</td>
<td>870.6</td>
<td>0.3010</td>
<td>0.8304</td>
</tr>
</tbody>
</table>

For constant \(c_v\), internal energy differences may be obtained by integrating \(du = c_v dT\). The energy transfer per unit mass may then be found as

\[
\begin{align*}
\Delta W_c/m &= c_v (T_2 - T_1) = 0.7176 (673.1 - 293) = 272.8 \text{ kJ/kg} \\
\Delta Q_H/m &= c_v (T_3 - T_2) = 0.7176 (2000 - 673.1) = 952.2 \text{ kJ/kg} \\
\Delta W_e/m &= c_v (T_3 - T_4) = 0.7176 (2000 - 870.6) = 810.5 \text{ kJ/kg} \\
\Delta Q_L/m &= c_v (T_4 - T_1) = 0.7176 (870.6 - 293) = 414.5 \text{ kJ/kg}
\end{align*}
\]