**Problem: 6-5**

**Given:** At the start of the compression process in a Diesel-cycle engine the air is at 530 R, 14.7 psia. The compression ratio is 16. The peak cycle temperature is equal to the 4000 R source temperature. The adiabatic compression and expansion processes are reversible.

**Find:** i) the energy transfer per unit mass for each process in the cycle

**Assume:** Even though high temperatures are involved, assume that specific heats are constants at their low-temperture values.

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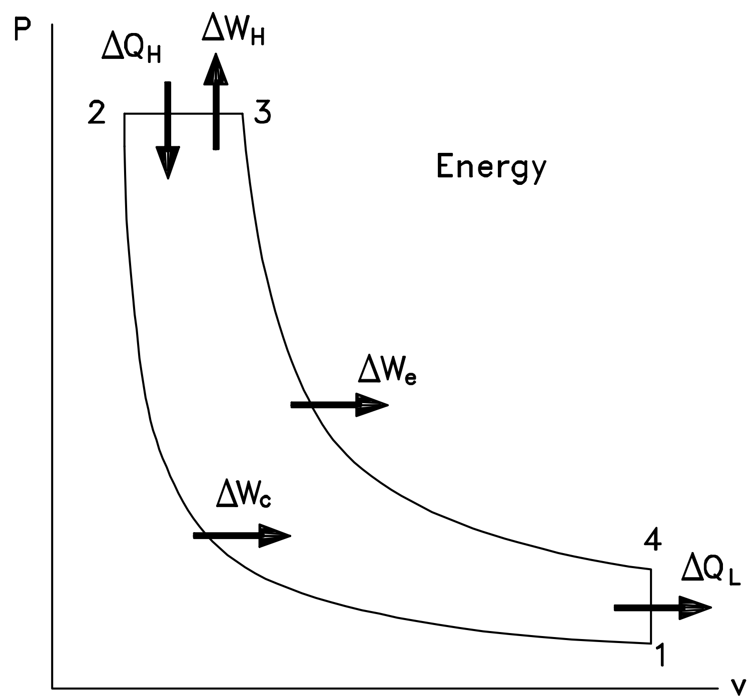



Solution:
The analysis begins with the evaluation of properties.

| State | $T$ $(K)$ | $P$ $(MPa)$ | $v$ $(m^3/kg)$ |
|---|---|---|---|
| 1 | 530 | 14.7 | |
| 2 | | | |
| 3 | 4000 | | |
| 4 | | | |

The specific volume at state (1) may be computed from the ideal-gas equation as:

$$\begin{aligned} v_1 &= R\ T_1/P_1 \\ &= \frac{53.35\ ft - lbf/(lbm - R)\ \times 530\ R}{14.7\ lbf/in^2\ \times\ 144\ in^2/ft^2} \\ &= 13.36\ ft^3/lbm \end{aligned}$$

An entropy balance for the reversible, adiabatic compression process gives $s_1 = s_2$. The compression ratio gives $v_1/v_2 = 16.0$ The value of $T_2$ may be found by integrating $T\ ds = du + P\ dv$ for the isentropic process from state (1) to state (2) to obtain:

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

Upon multiplying by $T_1$ and substituting numerical values

$$T_2 = 530 \times 16.0^{(1.4-1)} = 1606.7\ R$$

The pressure after compression may be found from the ideal gas equation as

$$P_2 = m_2\ R\ T_2/V_2$$

Since $m_2 = m_1 = (P_1V_1)/(R\ T_1)$ we may write

$$\begin{aligned} P_2 &= [(P_1V_1)/(R\ T_1)][R\ T_2/V_2] \\ &= P_1(V_1/V_2)(T_2/T_1) \\ &= 14.7 \times 16.0 \times (1606.7/530) \\ &= 713.0\ psia \end{aligned}$$

The specific volume at state (2) may be computed from the compression ratio as

$$v_2 = v_1/(v_1/v_2) = 13.36/16.0 = 0.835\ ft^3/lbm$$

Since $T_3$ is given as 4000 $R$ and $P_3 = P_2$, $v_3$ may be found as

$$\begin{aligned} v_3 &= R\ T_3/P_3 \\ &= \frac{53.35\ ft - lbf/(lbm - R)\ \times 4000\ R}{713.0\ lbf/in^2\ \times\ 144\ in^2/ft^2} \\ &= 2.078\ ft^3/lbm \end{aligned}$$

State (4) is fixed by $s_4 = s_3$ and $v_4 = v_1$. The value of $T_4$ can be found by integrating $T\ ds = du + P\ dV$ for the isentropic process from state (3) to state (4) to obtain

$$\frac{T_4}{T_3} = \left(\frac{v_4}{v_3}\right)^{1-k}$$

Since $v_4 = v_1$ we may substitute values for $v_1$ and $v_3$ to obtain

$$T_4 = 4000 \times (13.36/2.078)^{(1-1.4)} = 1900.2\ R$$

The pressure at state (4) is given by

$$P_4 = R\ T_4/v_4$$

Since $v_4 = v_1 = R\ T_1/P_1$ we may write

$$P_4 = (T_4/T_1)P_1 = (1900/530) \times 14.7 = 52.7\ psia$$

The table of property values may now be completed.

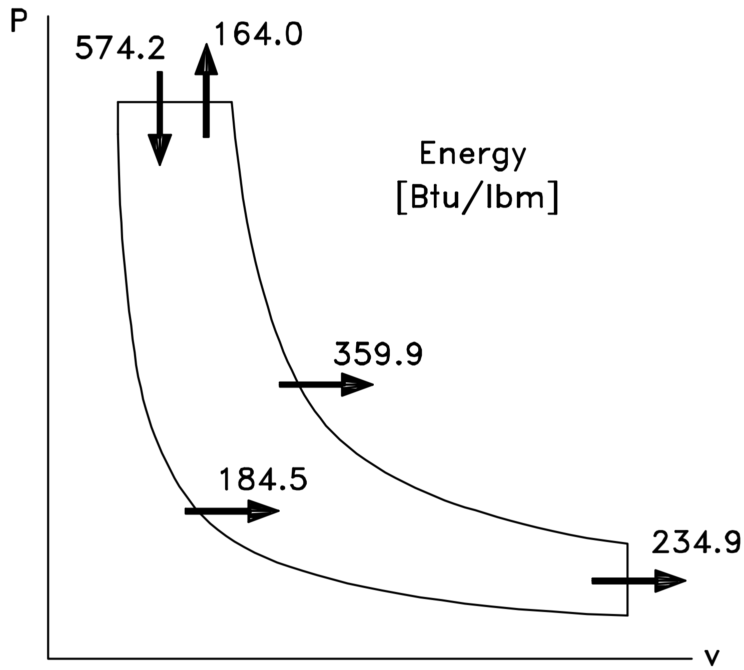
| **State** | $\boldsymbol{T}$ $\boldsymbol{(K)}$ | $\boldsymbol{P}$ $\boldsymbol{(MPa)}$ | $\boldsymbol{v}$ $\boldsymbol{(m^3/kg)}$ |
|---|---|---|---|
| 1 | 530 | 14.7 | 13.36 |
| 2 | 1606.7 | 713.0 | 0.835 |
| 3 | 4000 | 713.0 | 2.078 |
| 4 | 1900.2 | 52.7 | 13.36 |

For constant specific heats, the energy transfers per unit mass may be found as:

$$\begin{aligned}
\Delta W_c/m = &\quad c_v(T_2 - T_1) = 0.1714\ (1606.7 - 530) &= 184.5\ Btu/lbm \\
\Delta Q_H/m = &\quad c_p(T_3 - T_2) = 0.2399\ (4000 - 1606.7) &= 574.2\ Btu/lbm \\
\Delta W_e/m = &\quad c_v(T_3 - T_4) = 0.1714\ (4000 - 1900.2) &= 359.9\ Btu/lbm \\
\Delta Q_L/m = &\quad c_v(T_4 - T_1) = 0.1714\ (1900.2 - 530) &= 234.9\ Btu/lbm
\end{aligned}$$

For the heating process

$$\begin{aligned}
\Delta W_H/m &= P_o(v_3 - v_2) \\
&= \frac{713.0\ lbf/in^2 \times (2.078 - 0.835)\ ft^3/lbm \times 144\ in^2/ft^2}{778.2\ Btu/ft - lbf} \\
&= 164.0\ Btu/lbm
\end{aligned}$$


P
574.2
164.0
Energy
[Btu/lbm]
359.9
184.5
234.9
v