Problem: 6-5

Given: At the start of the compression process in a Diesel-cycle engine the air is at 530 R, 14.7 psia. The compression ratio is 16. The peak cycle temperature is equal to the 4000 R source temperature. The adiabatic compression and expansion processes are reversible.

Find: i) the energy transfer per unit mass for each process in the cycle

Assume: Even though high temperatures are involved, assume that specific heats are constants at their low-temperature values.

Solution: The analysis begins with the evaluation of properties.

<table>
<thead>
<tr>
<th>State</th>
<th>$T$ (K)</th>
<th>$P$ (MPa)</th>
<th>$v$ ($m^3/kg$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>530</td>
<td>14.7</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The specific volume at state (1) may be computed from the ideal-gas equation as:

\[ v_1 = \frac{R T_1}{P_1} \]

\[ = \frac{53.35 \text{ ft} - \text{lbf}/(\text{lbm} - R) \times 530 \text{ R}}{14.7 \text{ lbf}/\text{in}^2 \times 144 \text{ in}^2/\text{ft}^2} \]

\[ = 13.36 \text{ ft}^3/\text{lbm} \]

An entropy balance for the reversible, adiabatic compression process gives \( s_1 = s_2 \). The compression ratio gives \( v_1/v_2 = 16.0 \) The value of \( T_2 \) may be found by integrating \( T \, ds = du + P \, dv \) for the isentropic process from state (1) to state (2) to obtain:

\[ \frac{T_2}{T_1} = \left( \frac{v_1}{v_2} \right)^{k-1} \]

Upon multiplying by \( T_1 \) and substituting numerical values

\[ T_2 = 530 \times 16.0^{(1.4-1)} = 1606.7 \text{ R} \]

The pressure after compression may be found from the ideal gas equation as

\[ P_2 = m_2 \, R \, T_2/V_2 \]

Since \( m_2 = m_1 = (P_1V_1)/(R \, T_1) \) we may write

\[ P_2 = [ (P_1V_1)/(R \, T_1) ] [ R \, T_2/V_2 ] \]

\[ = P_1(V_1/V_2)(T_2/T_1) \]

\[ = 14.7 \times 16.0 \times (1606.7/530) \]

\[ = 713.0 \text{ psia} \]

The specific volume at state (2) may be computed from the compression ratio as

\[ v_2 = v_1/(v_1/v_2) = 13.36/16.0 = 0.835 \text{ ft}^3/\text{lbm} \]

Since \( T_3 \) is given as 4000 R and \( P_3 = P_2 \), \( v_3 \) may be found as

\[ v_3 = \frac{R \, T_3}{P_3} \]

\[ = \frac{53.35 \text{ ft} - \text{lbf}/(\text{lbm} - R) \times 4000 \text{ R}}{713.0 \text{ lbf}/\text{in}^2 \times 144 \text{ in}^2/\text{ft}^2} \]

\[ = 2.078 \text{ ft}^3/\text{lbm} \]
State (4) is fixed by \( s_4 = s_3 \) and \( v_4 = v_1 \). The value of \( T_4 \) can be found by integrating \( T \, ds = du + P \, dV \) for the isentropic process from state (3) to state (4) to obtain

\[
\frac{T_4}{T_3} = \left( \frac{v_4}{v_3} \right)^{1-k}
\]

Since \( v_4 = v_1 \) we may substitute values for \( v_1 \) and \( v_3 \) to obtain

\[
T_4 = 4000 \times (13.36/2.078)^{(1-1.4)} = 1900.2 \, R
\]

The pressure at state (4) is given by

\[
P_4 = R \, T_4/v_4
\]

Since \( v_4 = v_1 = R \, T_1/P_1 \) we may write

\[
P_4 = (T_4/T_1)P_1 = (1900/530) \times 14.7 = 52.7 \, psia
\]

The table of property values may now be completed.

<table>
<thead>
<tr>
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<td>13.36</td>
</tr>
<tr>
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<td>713.0</td>
<td>0.835</td>
</tr>
<tr>
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<td>4000</td>
<td>713.0</td>
<td>2.078</td>
</tr>
<tr>
<td>4</td>
<td>1900.2</td>
<td>52.7</td>
<td>13.36</td>
</tr>
</tbody>
</table>

For constant specific heats, the energy transfers per unit mass may be found as:

\[
\Delta W_c/m = c_v(T_2 - T_1) = 0.1714 \, (1606.7 - 530) = 184.5 \, Btu/lbm
\]

\[
\Delta Q_H/m = c_p(T_3 - T_2) = 0.2399 \, (4000 - 1606.7) = 574.2 \, Btu/lbm
\]

\[
\Delta W_e/m = c_v(T_3 - T_4) = 0.1714 \, (4000 - 1900.2) = 359.9 \, Btu/lbm
\]

\[
\Delta Q_L/m = c_v(T_4 - T_1) = 0.1714 \, (1900.2 - 530) = 234.9 \, Btu/lbm
\]

For the heating process

\[
\Delta W_H/m = P_o(v_3 - v_2) = \frac{713.0 \, lbf/in^2 \times (2.078 - 0.835) \, ft^3/lbm \times 144 \, in^2/ft^2}{778.2 \, Btu/ft \cdot lbf} = 164.0 \, Btu/lbm
\]