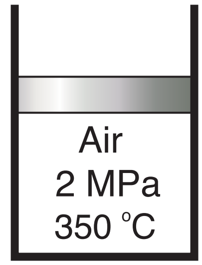
ME354
Thermodynamics 2

Quiz #1 - T01:

Name:	____________________
ID #:	____________________

Problem: A piston-cylinder device contains $\mathbf{0.15}\ \boldsymbol{kg}$ of air initially at $\mathbf{2}\ \boldsymbol{MPa}$ and $\mathbf{350}\ \boldsymbol{^\circ C}$. The air is first expanded isothermally to $\mathbf{500}\ \boldsymbol{kPa}$, then compressed polytropically with a polytropic exponent of $\mathbf{1.2}$ to the initial pressure, and finally compressed at the constant pressure to the initial state. Determine the boundary work for each process and the net work of the cycle.



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1. If air properties are calculated at $\mathbf{300}\ \boldsymbol{K}$, then from Table A-2a

$$R = 0.287\ kJ/kg \cdot K$$
$$k = 1.4$$

2. If air properties are calculated at $\mathbf{350}\ \boldsymbol{K}$, then from Table A-2b

$$R = c_p - c_v = 1.008 - 0.721 = 0.287\ kJ/kg \cdot K$$
$$k = 1.398$$

Since there is very little difference, I will proceed with case 1.

Isothermal Expansion

$$V_1 = \frac{mRT}{P_1} = \frac{(0.15\ kg)(0.287\ kJ/kg \cdot K)(350 + 273.15)\ K}{2000\ kPa \times \dfrac{1\ kJ}{1\ kPa \cdot m^3}} = 0.01341\ m^3$$

1 mark

$$V_2 = \frac{mRT}{P_2} = \frac{(0.15\ kg)(0.287\ kJ/kg \cdot K)(350 + 273.15)\ K}{500\ kPa \times \dfrac{1\ kJ}{1\ kPa \cdot m^3}} = 0.05365\ m^3$$

1 mark

The work for an isothermal process is given as

$$W_{1-2} = P_1 V_1 \ln\left(\frac{V_2}{V_1}\right) = (2000\ kPa)(0.01341\ m^3)\left(\frac{1\ kJ}{1\ kPa \cdot m^3}\right)\ln\left(\frac{0.05365}{0.01341}\right)$$
$$= 37.2\ kJ \Leftarrow$$

2 marks

Polytropic Compression

We know for a polytropic process that $Pv^n = constant$, which for a constant mass also implies that $PV^n = constant$.

$$P_2 V_2^n = P_3 V_3^n \longrightarrow (500\,kPa)(0.05365\,m^3)^{1.2} = (2000\,kPa)V_3^{1.2}$$

or

1 mark

$$V_3 = 0.0169\,m^3$$

$$W_{2-3} = \frac{P_3 V_3 - P_2 V_2}{1-n} = \frac{(2000\,kPa)(0.0169\,m^3) - (500\,kPa)(0.05364\,m^3)}{1-1.2}\left(\frac{1\,kJ}{1\,kPa \cdot m^3}\right)$$

$$= -34.9\,kJ \Leftarrow$$

2 marks

Constant Pressure Compression

The work at constant pressure is known as PdV work, therefore

$$W_{3-1} = P_3(V_1 - V_3) = (2000\,kPa)(0.01341\,m^3 - 0.0169\,m^3)\left(\frac{1\,kJ}{1\,kPa \cdot m^3}\right)$$

$$= -6.98\,kJ \Leftarrow$$

2 marks

Net Work

$$W_{net} = W_{1-2} + W_{2-3} + W_{3-1} = 37.2 + (-34.9) + (-6.98) = -4.68\,kJ \Leftarrow$$

1 mark