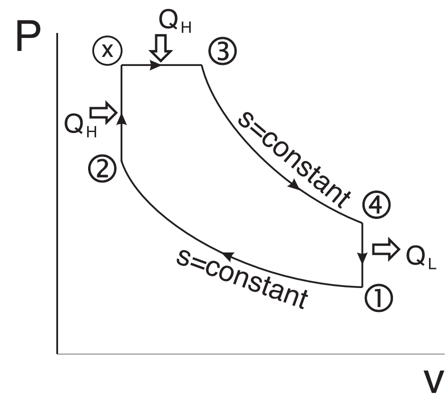
$\mathcal{ME}$354
Thermodynamics 2

Quiz #4 - T01:

Name: ____________________

ID #: ____________________

Problem: The compression ratio of an ideal dual cycle is 14. Air is at $100\ kPa$ and $300\ K$ at the beginning of the compression process and at $2200\ K$ at the end of the heat-addition process. Heat transfer to air takes place partly at constant volume and partly at constant pressure, and it amounts to $1520.4\ kJ/kg$. Assuming variable specific heats for air, determine:



(a) the fraction of heat transferred at constant volume and
(b) the thermal efficiency of the cycle.

Assumptions

1. air standard assumptions
2. $KE = PE \rightarrow 0$
3. air is an ideal gas with variable specific heats

Part a) The process from $1-2$ is isentropic compression. From Table A-17 at $T_1 = 300\ K$

$$u_1 = 214.07\ kJ/kg$$

$$v_{r_1} = 621.2$$

We can find the corrected properties at point 2 by:

$$v_{r_2} = \frac{v_2}{v_1} v_{r_1} = \frac{1}{r} v_{r_1} = \frac{1}{14}(621.2) = 44.37$$

and

$$T_2 = 823.14\ K$$

$$u_2 = 611.16\ kJ/kg$$

The process from 2-x and x-3 is a heat addition process. From Table A-17 at $T_3 = 2200\ K$

$$h_3 = 2503.2\ kJ/kg$$

$$v_{r_3} = 2.012$$

From a 1st law energy balance

$$q_{in} = q_{x-2,in} + q_{3-x,in} = (u_x - u_2) + (h_3 - h_x)$$

$$1520.4 = (u_x - 611.16) + (2503.2 - h_x)$$

By trial and error , from Table A-17, we see that the following conditions satisfy the energy balance

$$\begin{aligned} T_x &= 1300\ K \\ h_x &= 1395.97\ kJ/kg \\ u_x &= 1022.82\ kJ/kg \end{aligned}$$

and

$$q_{2-x,in} = u_x = u_2 = 1022.82 - 611.16 = 411.66\ kJ/kg$$

and

$$ratio = \frac{q_{2-x,in}}{q_{in}} = \frac{411.66}{1520.4} = 0.271 = 27.1\% \Leftarrow \textbf{part a)}$$

Part b)

To find the efficiency we need to determine q_{out}. First, determine the cutoff ratio between $x - 3$. For an ideal gas

$$P_3 = \frac{T_3}{v_3} = P_x = \frac{T_x}{v_x} \qquad \longrightarrow \qquad r_v = \frac{v_3}{v_x} = \frac{T_3}{T_x} = \frac{2200\ K}{1300\ K} = 1.692$$

For the isentropic process between $3 - 4$

$$\begin{aligned} v_{r_4} &= \frac{v_4}{v_3} v_{r_3} = \frac{v_4}{1.692 v_x} v_{r_3} = \frac{r}{1.692} v_{r_3} \\ &= \frac{14}{1.692}(2.012) = 16.648 \end{aligned}$$

From Table A-17

$$u_4 = 886.3\ kJ/kg$$

and

$$q_{out} = u_4 - u_1 = 886.3 - 214.07 = 672.23\ kJ/kg$$

The cycle efficiency is

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = \frac{672.23}{1520.4} = 0.558 = 55.8\%$$