12.16

A large stationary Brayton cycle gas-turbine power plant delivers a power output of 100 MW to an electric generator. The minimum temperature in the cycle is 300 K, and the maximum temperature is 1600 K. The minimum pressure in the cycle is 100 kPa, and the compressor pressure ratio is 14 to 1. Calculate the power output of the turbine. What fraction of the turbine output is required to drive the compressor? What is the thermal efficiency of the cycle?

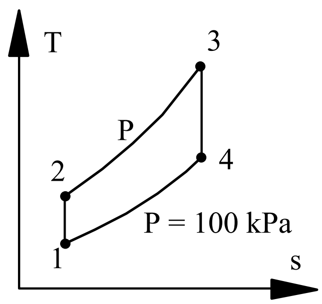
Solution:

Brayton cycle so this means:

Minimum T: $T_1 = 300$ K

Maximum T: $T_3 = 1600$ K

Pressure ratio: $P_2/P_1 = 14$



Solve using constant C_{P0}

Compression in compressor: $s_2 = s_1 \;\Rightarrow$ Implemented in Eq.8.32

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 300(14)^{0.286} = 638.1 \text{ K}$$

$$w_C = h_2 - h_1 = C_{P0}(T_2 - T_1) = 1.004\ (638.1 - 300) = 339.5 \text{ kJ/kg}$$

Expansion in turbine: $s_4 = s_3 \;\Rightarrow$ Implemented in Eq.8.32

$$T_4 = T_3\ (P_4/P_3)^{\frac{k-1}{k}} = 1600\ (1/14)^{0.286} = 752.2 \text{ K}$$

$$w_T = h_3 - h_4 = C_{P0}(T_3 - T_4) = 1.004\ (1600 - 752.2) = 851.2 \text{ kJ/kg}$$

$$w_{NET} = 851.2 - 339.5 = 511.7 \text{ kJ/kg}$$

Do the overall net and cycle efficiency

$$\dot{m} = \dot{W}_{NET}/w_{NET} = 100000/511.7 = 195.4 \text{ kg/s}$$

$$\dot{W}_T = \dot{m}w_T = 195.4 \times 851.2 = \mathbf{166.32\ MW}$$

$$w_C/w_T = 339.5/851.2 = \mathbf{0.399}$$

Energy input is from the combustor

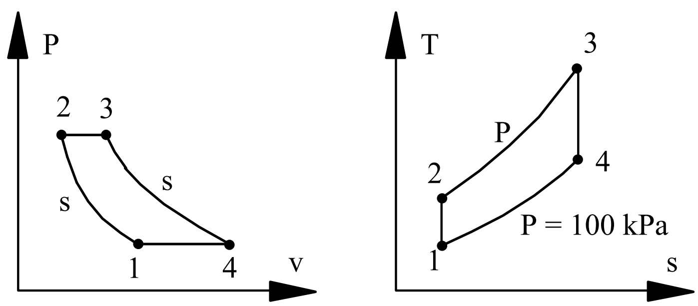
$$q_H = C_{P0}(T_3 - T_2) = 1.004\ (1600 - 638.1) = 965.7 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 511.7/965.7 = \mathbf{0.530}$$

12.17

Consider an ideal air-standard Brayton cycle in which the air into the compressor is at 100 kPa, 20°C, and the pressure ratio across the compressor is 12:1. The maximum temperature in the cycle is 1100°C, and the air flow rate is 10 kg/s. Assume constant specific heat for the air, value from Table A.5. Determine the compressor work, the turbine work, and the thermal efficiency of the cycle.

Solution:



Compression ratio

$$\frac{P_2}{P_1} = 12$$

Max temperature

$$T_3 = 1100^{o}C$$

$$\dot{m} = 10 \text{ kg/s}$$

The compression is reversible and adiabatic so constant s. From Eq.8.32

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 293.2(12)^{0.286} = 596.8 \text{ K}$$

Energy equation with compressor work in

$$w_C = -{}_1w_2 = C_{P0}(T_2 - T_1) = 1.004(596.8 - 293.2) = 304.8 \text{ kJ/kg}$$

The expansion is reversible and adiabatic so constant s. From Eq.8.32

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = 1373.2\left(\frac{1}{12}\right)^{0.286} = 674.7 \text{ K}$$

Energy equation with turbine work out

$$w_T = C_{P0}(T_3 - T_4) = 1.004(1373.2 - 674.7) = 701.3 \text{ kJ/kg}$$

Scale the work with the mass flow rate

$$\dot{W}_C = \dot{m}w_C = \mathbf{3048\ kW}, \qquad \dot{W}_T = \dot{m}w_T = \mathbf{7013\ kW}$$

Energy added by the combustion process

$$q_H = C_{P0}(T_3 - T_2) = 1.004(1373.2 - 596.8) = 779.5 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = (701.3 - 304.8)/779.5 = \mathbf{0.509}$$

12.18

Repeat Problem 12.17, but assume variable specific heat for the air, table A.7. Consider an ideal air-standard Brayton cycle in which the air into the compressor is at 100 kPa, 20°C, and the pressure ratio across the compressor is 12:1. The maximum temperature in the cycle is 1100°C, and the air flow rate is 10 kg/s. Assume constant specific heat for the air, value from Table A.5. Determine the compressor work, the turbine work, and the thermal efficiency of the cycle.

Solution:

From A.7: $h_1 = 293.6$ kJ/kg, $s^o_{T1} = 6.84597$ kJ/kg K

The compression is reversible and adiabatic so constant s. From Eq.8.28

$$s_2 = s_1 \;\Rightarrow\; s^o_{T2} = s^o_{T1} + R\ln(P_2/P_1) = 6.84597 + 0.287\ln 12 = 7.55914$$

$$\Rightarrow\; T_2 = 590 \text{ K},\quad h_2 = 597.2 \text{ kJ/kg}$$

Energy equation with compressor work in

$$w_C = -{}_1w_2 = h_2 - h_1 = 597.2 - 293.6 = 303.6 \text{ kJ/kg}$$

The expansion is reversible and adiabatic so constant s. From Eq.8.28

From A.7: $h_3 = 1483.1$ kJ/kg, $s^o_{T3} = 8.50554$ kJ/kg K

$$s_4 = s_3 \;\Rightarrow\; s^o_{T4} = s^o_{T3} + R\ln(P_4/P_3) = 8.50554 + 0.287\ln(1/12) = 7.79237$$

$$\Rightarrow\; T_4 = 734.8 \text{ K},\quad h_4 = 751.1 \text{ kJ/kg}$$

Energy equation with turbine work out

$$w_T = h_3 - h_4 = 1483.1 - 751.1 = 732 \text{ kJ/kg}$$

Scale the work with the mass flow rate

$$\Rightarrow\; \dot{W}_C = \dot{m}w_C = \mathbf{3036\ kW},\qquad \dot{W}_T = \dot{m}w_T = \mathbf{7320\ kW}$$

Energy added by the combustion process

$$q_H = h_3 - h_2 = 1483.1 - 597.2 = 885.9 \text{ kJ/kg}$$

$$w_{NET} = w_T - w_C = 732 - 303.6 = 428.4 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 428.4/885.9 = \mathbf{0.484}$$

12.29

A two-stage air compressor has an intercooler between the two stages as shown in Fig. P12.29. The inlet state is 100 kPa, 290 K, and the final exit pressure is 1.6 MPa. Assume that the constant pressure intercooler cools the air to the inlet temperature, $T_3 = T_1$. It can be shown that the optimal pressure, $P_2 = (P_1P_4)^{1/2}$, for minimum total compressor work. Find the specific compressor works and the intercooler heat transfer for the optimal P_2.

Solution:

Optimal intercooler pressure $P_2 = \sqrt{100 \times 1600} = 400$ kPa

1: $h_1 = 290.43$ kJ/kg, $s^o_{T1} = 6.83521$ kJ/kg K

C.V. C1: $w_{C1} = h_2 - h_1$, $s_2 = s_1$ leading to Eq.8.28

$\Rightarrow$ $s^o_{T2} = s^o_{T1} + R \ln(P_2/P_1) = 6.83521 + 0.287 \ln 4 = 7.2331$ kJ/kg K

$\Rightarrow$ $T_2 = 430.3$ K, $h_2 = 432.05$ kJ/kg

$w_{C1} = 432.05 - 290.43 =$ **141.6 kJ/kg**

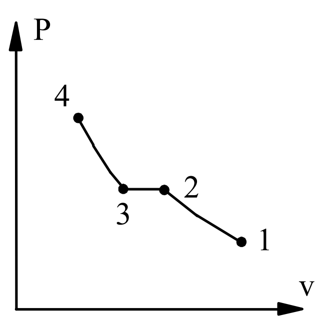
C.V. Cooler: $T_3 = T_1$ $\Rightarrow$ $h_3 = h_1$

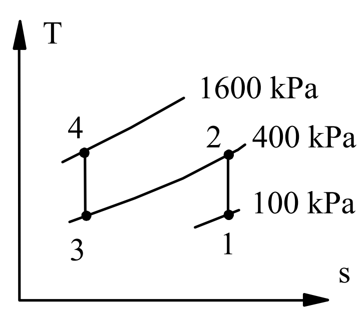
$q_{OUT} = h_2 - h_3 = h_2 - h_1 = w_{C1} =$ **141.6 kJ/kg**

C.V. C2: $T_3 = T_1$, $s_4 = s_3$ and since $s^o_{T3} = s^o_{T1}$, $P_4/P_3 = P_2/P_1$

$\Rightarrow$ $s^o_{T4} = s^o_{T3} + R \ln(P_4/P_3) = s^o_{T2}$, so we have $T_4 = T_2$

Thus we get $w_{C2} = w_{C1} =$ **141.6 kJ/kg**

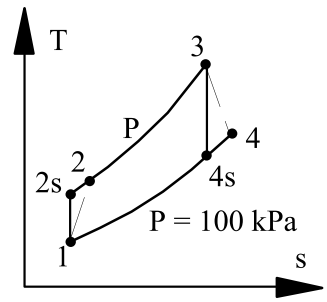




12.36

Repeat Problem 12.16, but assume that the compressor has an isentropic efficiency of 85% and the turbine an isentropic efficiency of 88%.

Solution:



Brayton cycle so this means:

Minimum T: $T_1 = 300$ K

Maximum T: $T_3 = 1600$ K

Pressure ratio: $P_2/P_1 = 14$

Solve using constant C_{P0}

Ideal compressor: $s_2 = s_1 \Rightarrow$ Implemented in Eq.8.32

$$T_{2s} = T_1(P_2/P_1)^{\frac{k-1}{k}} = 300(14)^{0.286} = 638.1 \text{ K}$$

$$w_{Cs} = h_2 - h_1 = C_{P0}(T_2 - T_1) = 1.004\ (638.1 - 300) = 339.5 \text{ kJ/kg}$$

Actual compressor

$$\Rightarrow\ w_C = w_{SC}/\eta_{SC} = 339.5/0.85 = 399.4 \text{ kJ/kg} = C_{P0}(T_2\text{-}T_1)$$

$$\Rightarrow\ T_2 = T_1 + w_c/C_{P0} = 300 + 399.4/1.004 = 697.8 \text{ K}$$

Ideal turbine: $s_4 = s_3 \Rightarrow$ Implemented in Eq.8.32

$$T_{4s} = T_3(P_4/P_3)^{\frac{k-1}{k}} = 1600\ (1/14)^{0.286} = 752.2 \text{ K}$$

$$w_{Ts} = h_3 - h_4 = C_{P0}(T_3 - T_4) = 1.004\ (1600 - 752.2) = 851.2 \text{ kJ/kg}$$

Actual turbine

$$\Rightarrow w_T = \eta_{ST}\ w_{ST} = 0.88 \times 851.2 = 749.1 \text{ kJ/kg} = C_{P0}(T_3\text{-}T_4)$$

$$\Rightarrow T_4 = T_3 - w_T/C_{P0} = 1600 - 749.1/1.004 = 853.9 \text{ K}$$

Do the overall net and cycle efficiency

$$w_{NET} = w_T - w_C = 749.1 - 399.4 = 349.7 \text{ kJ/kg}$$

$$\dot{m} = \dot{W}_{NET}/w_{NET} = 100000/349.7 = 286.0 \text{ kg/s}$$

$$\dot{W}_T = \dot{m}w_T = 286.0\times749.1 = \mathbf{214.2\ MW}$$

$$w_C/w_T = 399.4/749.1 = \mathbf{0.533}$$

Energy input is from the combustor

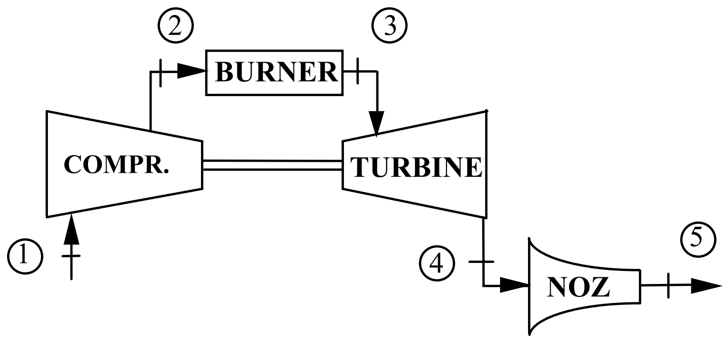
$$q_H = C_{P0}(T_3 - T_2) = 1.004(1600 - 697.8) = 905.8 \text{ kJ/kg}$$

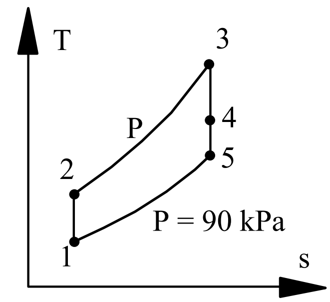
$$\eta_{TH} = w_{NET}/q_H = 349.7/905.8 = \mathbf{0.386}$$

12.43

Consider an ideal air-standard cycle for a gas-turbine, jet propulsion unit, such as that shown in Fig. 12.9. The pressure and temperature entering the compressor are 90 kPa, 290 K. The pressure ratio across the compressor is 14 to 1, and the turbine inlet temperature is 1500 K. When the air leaves the turbine, it enters the nozzle and expands to 90 kPa. Determine the velocity of the air leaving the nozzle.

Solution:





C.V. Compressor: Reversible and adiabatic $s_2 = s_1$ From Eq.8.25, 8.32

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 290\ (14)^{0.2857} = 616.4\ \text{K}$$

$$w_C = h_2 - h_1 \approx C_{P0}\ (T_2 - T_1) = 1.004\ (616.4 - 290) = 327.7\ \text{kJ/kg}$$

C.V. Turbine: $w_T = h_3 - h_4 = w_C$ and $s_4 = s_3 \Rightarrow$

$$T_4 = T_3 - w_C/C_{P0} = 1500 - 327.7/1.004 = 1173.6\ \text{K}$$

C.V. Nozzle: $s_5 = s_4 = s_3$ so from Eq.8.32

$$T_5 = T_3\left(\frac{P_5}{P_3}\right)^{\frac{k-1}{k}} = 1500\left(\frac{90}{1260}\right)^{0.2857} = 705.7\ \text{K}$$

Now the energy equation

$$(1/2)\mathbf{V}_5^2 = h_4 - h_5 \approx C_{P0}\ (T_4 - T_5) = 1.004\ (1173.6 - 705.7) = 469.77\ \text{kJ/kg}$$

$$\Rightarrow \quad \mathbf{V}_5 = \sqrt{2 \times 1000 \times 469.77} = \mathbf{969\ m/s}$$

12.44

Solve the previous problem using the air tables.

Consider an ideal air-standard cycle for a gas-turbine, jet propulsion unit, such as that shown in Fig. 12.9. The pressure and temperature entering the compressor are 90 kPa, 290 K. The pressure ratio across the compressor is 14 to 1, and the turbine inlet temperature is 1500 K. When the air leaves the turbine, it enters the nozzle and expands to 90 kPa. Determine the pressure at the nozzle inlet and the velocity of the air leaving the nozzle.

C.V. Compressor: Reversible and adiabatic $s_2 = s_1$ From Eq.8.28

$$\Rightarrow \quad s^o_{T2} = s^o_{T1} + R \ln(P_2/P_1) = 6.83521 + 0.287 \ln 14 = 7.59262 \text{ kJ/kg K}$$

From A.7 $h_2 = 617.2$ kJ/kg, $T_2 = 609.4$ K

$$w_C = h_2 - h_1 = 617.2 - 290.43 = 326.8 \text{ kJ/kg}$$

C.V. Turbine: $w_T = h_3 - h_4 = w_C$ and $s_4 = s_3 \Rightarrow$

$$h_4 = h_3 - w_C = 1635.8 - 326.8 = 1309 \text{ kJ/kg}$$

$$\Rightarrow \quad s^o_{T4} = 8.37142 \text{ kJ/kg K}, \quad T_4 = 1227 \text{ K}$$

$$P_4 = P_3 \exp[(s^o_{T4} - s^o_{T3})/R] = 1260 \exp[\,(8.37142 - 8.61208)/0.287\,]$$

$$= 1260 \exp(-0.83854) = \mathbf{544.8\ kPa}$$

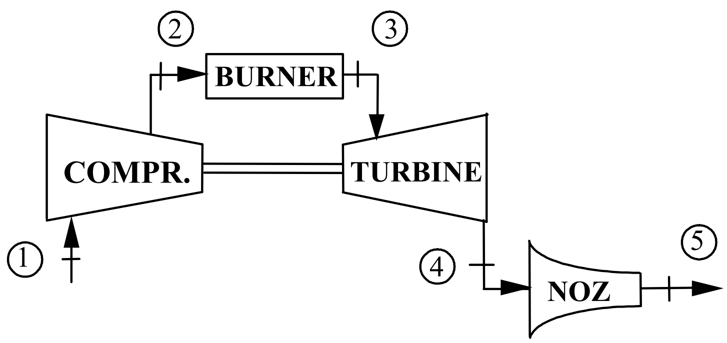
C.V. Nozzle: $s_5 = s_4 = s_3$ so from Eq.8.28

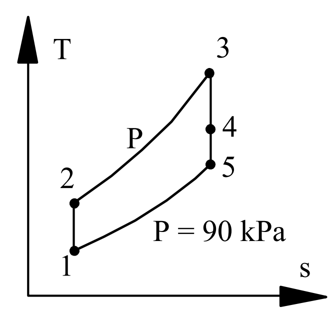
$$\Rightarrow \quad s^o_{T5} = s^o_{T3} + R \ln(P_5/P_3) = 8.61208 + 0.287 \ln (1/14) = 7.85467 \text{ kJ/kgK}$$

=> From A.7 $T_5 = 778$ K, $h_5 = 798.2$ kJ/kg

Now the energy equation

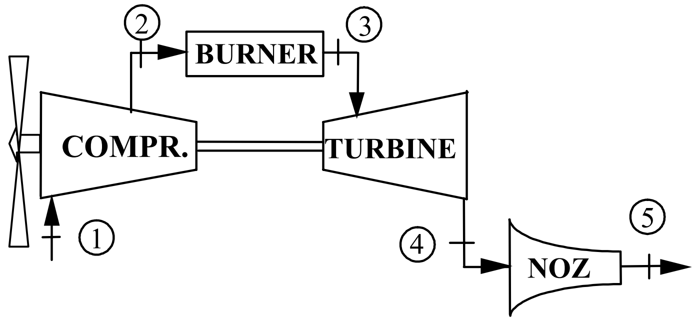
$$(1/2)\mathbf{V}_5^2 = h_4 - h_5 = 510.8 \quad \Rightarrow \quad \mathbf{V}_5 = \sqrt{2 \times 1000 \times 510.8} = \mathbf{1011\ m/s}$$

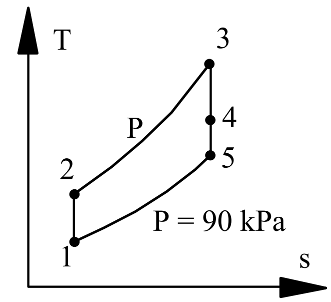




12.47

Consider a turboprop engine where the turbine powers the compressor and a propeller. Assume the same cycle as in Problem 12.43 with a turbine exit temperature of 900 K. Find the specific work to the propeller and the exit velocity.





C.V. Compressor: Reversible and adiabatic $s_2 = s_1$ From Eq.8.25, 8.32

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 290\ (14)^{0.2857} = 616.4\ \text{K}$$

$$w_C = h_2 - h_1 \approx C_{P0}\ (T_2 - T_1) = 1.004\ (616.4 - 290) = 327.7\ \text{kJ/kg}$$

C.V. Turbine: $w_T = h_3 - h_4 = w_C + w_{prop}$ and $s_4 = s_3 \Rightarrow$

$$w_{prop} = C_{P0}\ (T_3 - T_4) - w_C = 1.004(1500 - 900) - 327.7 = 274.7\ \text{kJ/kg}$$

C.V. Nozzle: $s_5 = s_4 = s_3$ so from Eq.8.32

$$T_5 = T_3\left(\frac{P_5}{P_3}\right)^{\frac{k-1}{k}} = 1500\left(\frac{90}{1260}\right)^{0.2857} = 705.7\ \text{K}$$

Now the energy equation

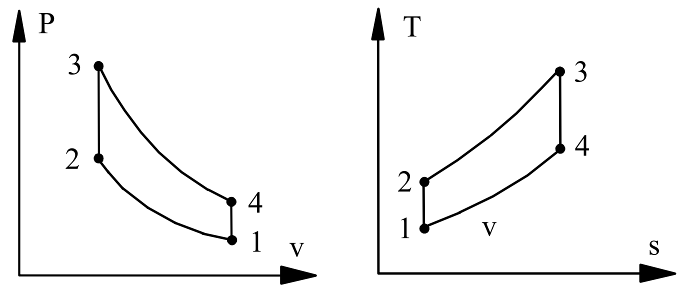
$$(1/2)\mathbf{V}_5^2 = h_4 - h_5 \approx C_{P0}\ (T_4 - T_5) = 1.004\ (900 - 705.7) = 195.08\ \text{kJ/kg}$$

$$\Rightarrow \quad \mathbf{V}_5 = \sqrt{2 \times 1000 \times 195.08} = \mathbf{625\ m/s}$$

12.67

To approximate an actual spark-ignition engine consider an air-standard Otto cycle that has a heat addition of 1800 kJ/kg of air, a compression ratio of 7, and a pressure and temperature at the beginning of the compression process of 90 kPa, 10°C. Assuming constant specific heat, with the value from Table A.5, determine the maximum pressure and temperature of the cycle, the thermal efficiency of the cycle and the mean effective pressure.

Solution:



Compression: Reversible and adiabatic so constant s from Eq.8.33-34

$$P_2 = P_1(v_1/v_2)^k = 90(7)^{1.4} = 1372 \text{ kPa}$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 283.2 \times (7)^{0.4} = 616.6 \text{ K}$$

Combustion: constant volume

$$T_3 = T_2 + q_H/C_{V0} = 616.6 + 1800/0.717 = \mathbf{3127\ K}$$

$$P_3 = P_2T_3/T_2 = 1372 \times 3127 / 616.6 = \mathbf{6958\ kPa}$$

Efficiency and net work

$$\eta_{TH} = 1 - T_1/T_2 = 1 - 283.2/616.5 = \mathbf{0.541}$$

$$w_{net} = \eta_{TH} \times q_H = 0.541 \times 1800 = 973.8 \text{ kJ/kg}$$

Displacement and P_{meff}

$$v_1 = RT_1/P_1 = (0.287 \times 283.2)/90 = 0.9029 \text{ m}^3\text{/kg}$$

$$v_2 = (1/7)\, v_1 = 0.1290 \text{ m}^3\text{/kg}$$

$$P_{meff} = \frac{w_{NET}}{v_1 - v_2} = \frac{973.8}{0.9029 - 0.129} = \mathbf{1258\ kPa}$$

12.74

Repeat Problem 12.67, but assume variable specific heat. The ideal gas air tables, Table A.7, are recommended for this calculation (or the specific heat from Fig. 5.10 at high temperature).

Solution:

Table A.7 is used with interpolation.

$T_1 = 283.2$ K, $u_1 = 202.3$ kJ/kg, $s^o_{T1} = 6.8113$ kJ/kg K

Compression 1 to 2: $s_2 = s_1 \Rightarrow$ From Eq.8.28

$$0 = s^o_{T2} - s^o_{T1} - R \ln(P_2/P_1) = s^o_{T2} - s^o_{T1} - R \ln(T_2v_1/T_1v_2)$$

$$s^o_{T2} - R \ln(T_2/T_1) = s^o_{T1} + R \ln(v_1/v_2) = 6.8113 + 0.287 \ln 7 = 7.3698$$

This becomes trial and error so estimate first at 600 K and use A.7.1.

$LHS_{600} = 7.5764 - 0.287 \ln(600/283.2) = 7.3609$ (too low)

$LHS_{620} = 7.6109 - 0.287 \ln(620/283.2) = 7.3860$ (too high)

Interpolate to get: $T_2 = 607.1$ K, $u_2 = 440.5$ kJ/kg

$\Rightarrow -{}_1w_2 = u_2 - u_1 = 238.2$ kJ/kg,

$u_3 = 440.5 + 1800 = 2240.5 \Rightarrow T_3 =$ **2575.8 K** , $s^o_{T3} = 9.2859$ kJ/kgK

$P_3 = 90 \times 7 \times 2575.8 / 283.2 =$ **5730 kPa**

Expansion 3 to 4: $s_4 = s_3 \Rightarrow$ From Eq.8.28 as before

$$s^o_{T4} - R \ln(T_4/T_3) = s^o_{T3} + R \ln(v_3/v_4) = 9.2859 + 0.287 \ln(1/7) = 8.7274$$

This becomes trial and error so estimate first at 1400 K and use A.7.1.

$LHS_{1400} = 8.5289 - 0.287 \ln(1400/2575.8) = 8.7039$ (too low)

$LHS_{1450} = 8.5711 - 0.287 \ln(1450/2575.8) = 8.7360$ (too high)

Interpolation $\Rightarrow T_4 = 1436.6$ K, $u_4 = 1146.9$ kJ/kg

${}_3w_4 = u_3 - u_4 = 2240.5 - 1146.9 = 1093.6$ kJ/kg

Net work, efficiency and mep

➔ $w_{net} = {}_3w_4 + {}_1w_2 = 1093.6 - 238.2 = 855.4$ kJ/kg

$\eta_{TH} = w_{net} / q_H = 855.4 / 1800 =$ **0.475**

$v_1 = RT_1/P_1 = (0.287 \times 283.2)/90 = 0.9029\ m^3/kg$

$v_2 = (1/7)\ v_1 = 0.1290\ m^3/kg$

$$P_{meff} = \frac{w_{net}}{v_1 - v_2} = 855.4 / (0.9029 - 0.129) = \mathbf{1105\ kPa}$$

12.81

It is found experimentally that the power stroke expansion in an internal combustion engine can be approximated with a polytropic process with a value of the polytropic exponent *n* somewhat larger than the specific heat ratio *k*. Repeat Problem 12.67 but assume that the expansion process is reversible and polytropic (instead of the isentropic expansion in the Otto cycle) with *n* equal to 1.50.

See solution to 12.67 except for process 3 to 4.

$T_3 =$ **3127 K**, $P_3 =$ **6.958 MPa**

$v_3 = RT_3/P_3 = v_2 = 0.129\ m^3/kg$, $v_4 = v_1 = 0.9029\ m^3/kg$

Process: $Pv^{1.5} =$ constant.

$P_4 = P_3(v_3/v_4)^{1.5} = 6958\ (1/7)^{1.5} = 375.7$ kPa

$T_4 = T_3(v_3/v_4)^{0.5} = 3127(1/7)^{0.5} = 1181.9$ K

$${}_1w_2 = \int Pdv = \frac{R}{1\text{-}1.4}(T_2 - T_1) = \frac{0.287}{-0.4}(606.6 - 283.15) = -239.3 \text{ kJ/kg}$$

$${}_3w_4 = \int Pdv = R(T_4 - T_3)/(1 - 1.5)$$

$$= -0.287(1181.9\text{-}3127)/0.5 = 1116.5 \text{ kJ/kg}$$

$w_{NET} = 1116.5 - 239.3 = 877.2$ kJ/kg

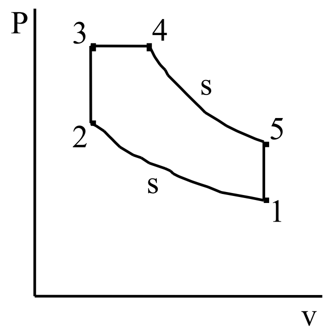
$\eta_{CYCLE} = w_{NET}/q_H = 877.2/1800 = \mathbf{0.487}$

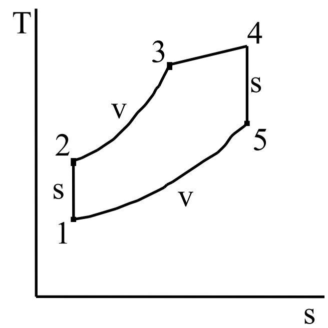
$P_{meff} = \dfrac{w_{net}}{v_1 - v_2} = 877.2/(0.9029 - 0.129) =$ **1133 kPa**

Note a smaller w_{NET}, η_{CYCLE}, P_{meff} compared to an ideal cycle.

12.82

In the Otto cycle all the heat transfer q_H occurs at constant volume. It is more realistic to assume that part of q_H occurs after the piston has started its downward motion in the expansion stroke. Therefore, consider a cycle identical to the Otto cycle, except that the first two-thirds of the total q_H occurs at constant volume and the last one-third occurs at constant pressure. Assume that the total q_H is 2100 kJ/kg, that the state at the beginning of the compression process is 90 kPa, 20°C, and that the compression ratio is 9. Calculate the maximum pressure and temperature and the thermal efficiency of this cycle. Compare the results with those of a conventional Otto cycle having the same given variables.





$P_1 = 90$ kPa, $T_1 = 20^oC$

$r_V = v_1/v_2 = 7$

a) $q_{23} = (2/3) \times 2100 = 1400$ kJ/kg;

$q_{34} = 2100/3 = 700$ kJ/kg

b) $P_2 = P_1(v_1/v_2)^k = 90(9)^{1.4} = 1951$ kPa

$T_2 = T_1(v_1/v_2)^{k-1} = 293.15(9)^{0.4} = 706$ K

$T_3 = T_2 + q_{23}/C_{V0} = 706 + 1400/0.717 = \mathbf{2660\ K}$

$P_3 = P_2T_3/T_2 = 1951(2660/706) = \mathbf{7350.8\ kPa} = P_4$

$T_4 = T_3 + q_{34}/C_{P0} = 2660 + 700/1.004 = 3357$ K

$$\frac{v_5}{v_4} = \frac{v_1}{v_4} = \frac{P_4}{P_1} \times \frac{T_1}{T_4} = \frac{7350.8}{90} \times \frac{293.15}{3357} = 7.131$$

$T_5 = T_4(v_4/v_5)^{k-1} = 3357(1/7.131)^{0.4} = 1530$ K

$q_L = C_{V0}(T_5\text{-}T_1) = 0.717(1530 - 293.15) = 886.2$ kJ/kg

$\eta_{TH} = 1 - q_L/q_H = 1 - 886.2/2100 = \mathbf{0.578}$

Std. Otto Cycle: $\eta_{TH} = 1 - (9)^{-0.4} = \mathbf{0.585}$, small difference

12.86

A diesel engine has a compression ratio of 20:1 with an inlet of 95 kPa, 290 K, state 1, with volume 0.5 L. The maximum cycle temperature is 1800 K. Find the maximum pressure, the net specific work and the thermal efficiency.

Solution:

Compression process (isentropic) from Eqs.8.33-34

$T_2 = T_1(v_1 / v_2)^{k-1} = 290 \times 20^{0.4} = 961$ K

$P_2 = 95 \times (20)^{1.4} = 6297.5$ kPa ;

$v_2 = v_1/20 = RT_1/(20\ P_1) = 0.043805\ m^3/kg$

$-{}_1w_2 = u_2 - u_1 \approx C_{vo}(\ T_2 - T_1) = 0.717\ (961 - 290) = 481.1$ kJ/kg

Combustion at constant P which is the maximum presssure

$P_3 = P_2 =$ **6298 kPa** ;

$v_3 = v_2\ T_3 / T_2 = 0.043805 \times 1800/961 = 0.08205\ m^3/kg$

${}_2w_3 = P\ (v_3 - v_2) = 6298 \times (0.08215 - 0.043805) = 241.5$ kJ/kg

${}_2q_3 = u_3 - u_2 + {}_2w_3 = h_3 - h_2 = C_{po}(T_3 - T_2)$

$= 1.004(1800 - 961) = 842.4$ kJ/kg

Expansion process (isentropic) from Eq.8.33

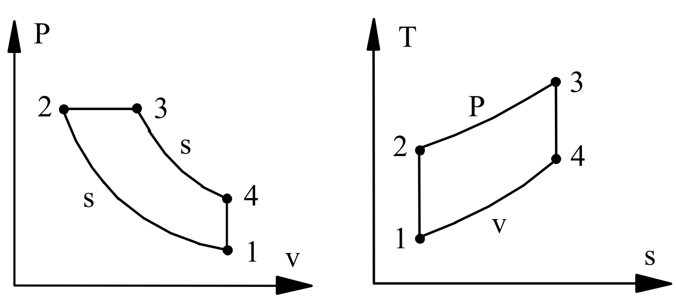
$T_4 = T_3(\ v_3 / v_4)^{0.4} = 1800\ (0.08205 / 0.8761)^{0.4} = 698$ K

${}_3w_4 = u_3 - u_4 \approx C_{vo}(T_3 - T_4) = 0.717\ (1800 - 698) = 790.1$ kJ/kg

Cycle net work and efficiency

$w_{net} = {}_2w_3 + {}_3w_4 + {}_1w_2 = 241.5 + 790.1 - 481.1 =$ **550.5 kJ/kg**

$\eta = w_{net} / q_H = 550.5/ 842.4 =$ **0.653**



12.89

At the beginning of compression in a diesel cycle T = 300 K, P = 200 kPa and after combustion (heat addition) is complete T = 1500 K and P = 7.0 MPa. Find the compression ratio, the thermal efficiency and the mean effective pressure.

Solution:

Standard Diesel cycle. See P-v and T-s diagrams for state numbers.

Compression process (isentropic) from Eqs.8.33-8.34

$$P_2 = P_3 = 7000 \text{ kPa} \;\; => \;\; v_1 / v_2 = (P_2/P_1)^{1/k} = (7000 / 200)^{0.7143} = \mathbf{12.67}$$

$$T_2 = T_1(P_2 / P_1)^{(k-1)/k} = 300(7000 / 200)^{0.2857} = 828.4 \text{ K}$$

Expansion process (isentropic) first get the volume ratios

$$v_3 / v_2 = T_3 / T_2 = 1500 / 828.4 = 1.81$$

$$v_4 / v_3 = v_1 / v_3 = (v_1 / v_2)(v_2 / v_3) = 12.67 / 1.81 = 7$$

The exhaust temperature follows from Eq.8.33

$$T_4 = T_3(v_3 / v_4)^{k-1} = 1500 \; (1 / 7)^{0.4} = 688.7 \text{ K}$$

$$q_L = C_{vo}(T_4 - T_1) = 0.717(688.7 - 300) = 278.5 \text{ kJ/kg}$$

$$q_H = h_3 - h_2 \approx C_{po}(T_3 - T_2) = 1.004(1500 - 828.4) = 674 \text{ kJ/kg}$$

Overall performance

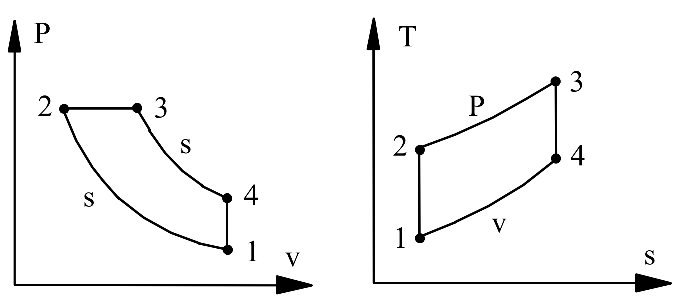
$$\eta = 1 - q_L / q_H = 1- 278.5 / 674 = \mathbf{0.587}$$

$$w_{net} = q_{net} = q_H - q_L = 674 - 278.5 = 395.5 \text{ kJ/kg}$$

$$v_{max} = v_1 = R\, T_1 / P_1 = 0.287 \times 300 / 200 = 0.4305 \text{ m}^3\text{/kg}$$

$$v_{min} = v_{max} / (v_1 / v_2) = 0.4305 / 12.67 = 0.034 \text{ m}^3\text{/kg}$$

$$P_{meff} = \frac{w_{net}}{v_{max} - v_{min}} = 395.5 / (0.4305 - 0.034) = \mathbf{997\ kPa}$$



Remark: This is a too low compression ratio for a practical diesel cycle.

12.94

Consider an ideal air-standard diesel cycle in which the state before the compression process is 95 kPa, 290 K, and the compression ratio is 20. Find the thermal efficiency for a maximum temperature of 2200 K

Solution:

Diesel cycle: $P_1 = 95$ kPa, $T_1 = 290$ K, $CR = v_1/v_2 = 20$,

$$v_1 = 0.287 \times 290/95 = 0.8761 \text{ m}^3\text{/kg} = v_4 = CR\ v_2,$$

Compression process (isentropic) from Eqs.8.33-34

$$T_2 = T_1(v_1 / v_2)^{k-1} = 290 \times 20^{0.4} = 961.2 \text{ K}$$

Combustion at constant P which is the maximum pressure

$$v_3 = v_2\ T_3 / T_2 = \frac{0.8761}{20} \times \frac{2200}{961.2} = 0.10026 \text{ m}^3\text{/kg}$$

Expansion process (isentropic) from Eq.8.33

$$T_4 = T_3(\ v_3 / v_4)^{0.4} = 2200\ (0.10026 / 0.8761)^{\ 0.4} = 924.4 \text{ K}$$

Cycle net work and efficiency

$$\eta_{TH} = 1 - \frac{T_4 - T_1}{k(T_3 - T_2)} = 1 - \frac{924.4 - 290}{1.4\ (2200 - 961.2)} = \mathbf{0.634}$$

12.116

A Brayton cycle has a compression ratio of 15:1 with a high temperature of 1600 K and an inlet state of 290 K, 100 kPa. Use cold air properties to find the specific net work output and the second law efficiency if we neglect the "value" of the exhaust flow.

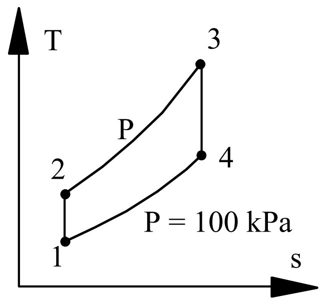
Brayton cycle so this means:

Minimum T: $T_1 = 290$ K

Maximum T: $T_3 = 1600$ K

Pressure ratio: $P_2/P_1 = 15$

Solve using constant C_{P0}



Compression in compressor: $s_2 = s_1 \Rightarrow$ Implemented in Eq.8.32

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 290(15)^{0.286} = 628.65 \text{ K}$$

Energy input is from the combustor

$$q_H = h_3 - h_2 = C_{P0}(T_3 - T_2) = 1.004\ (1600 - 628.65) = 975.2 \text{ kJ/kg}$$

Do the overall cycle efficiency and net work

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_{net}}{q_H} = 1 - r_p^{-(k-1)/k} = 1 - 15^{-0.4/1.4} = \mathbf{0.5387}$$

$$w_{NET} = \eta\ q_H = 0.5387 \times 975.2 = 525.34 \text{ kJ/kg}$$

Notice the q_H does not come at a single T so neglecting external irreversibility we get

$$\Phi_{qH} = \text{increase in flow exergy} = \psi_3 - \psi_2 = h_3 - h_2 - T_o(s_3 - s_2)$$

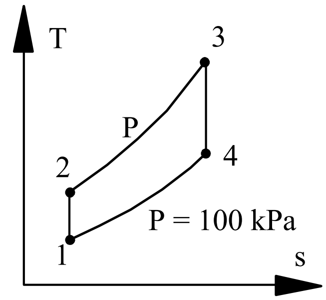
$$= q_H - T_o\ C_P \ln(T_3/T_2) = 975.2 - 298.15 \times 1.004 \ln\left(\frac{1600}{628.65}\right)$$

$$= 695.56 \text{ kJ/kg}$$

$$\eta_{II} = \frac{w_{net}}{\psi_3 - \psi_2} = \frac{525.34}{695.56} = \mathbf{0.755}$$

12.117

Reconsider the previous problem and find the second law efficiency if you do consider the "value" of the exhaust flow.



Brayton cycle so this means:

Minimum T: $T_1 = 290$ K

Maximum T: $T_3 = 1600$ K

Pressure ratio: $P_2/P_1 = 15$

Solve using constant C_{P0}

Compression in compressor: $s_2 = s_1 \Rightarrow$ Implemented in Eq.8.32

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 290(15)^{0.286} = 628.65 \text{ K}$$

Energy input is from the combustor

$$q_H = h_3 - h_2 = C_{P0}(T_3 - T_2) = 1.004\ (1600 - 628.65) = 975.2 \text{ kJ/kg}$$

Do the overall cycle thermal efficiency and net work

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_{net}}{q_H} = 1 - r_p^{-(k-1)/k} = 1 - 15^{-0.4/1.4} = \mathbf{0.5387}$$

$$w_{NET} = \eta\ q_H = 0.5387 \times 975.2 = 525.34 \text{ kJ/kg}$$

Notice the q_H does not come at a single T so neglecting external irreversibility we get

$$\Phi_{qH} = \text{increase in flow exergy} = \psi_3 - \psi_2 = h_3 - h_2 - T_o(s_3 - s_2)$$

$$= q_H - T_o\ C_P \ln(T_3/T_2) = 975.2 - 298.15 \times 1.004 \ln\left(\frac{1600}{628.65}\right)$$

$$= 695.56 \text{ kJ/kg}$$

$$T_4 = T_3\ (P_4/P_3)^{\frac{k-1}{k}} = 1600\ (1/15)^{0.286} = 738 \text{ K}$$

$$\Phi_{qL} = \psi_4 - \psi_1 = h_4 - h_1 - T_o(s_4 - s_1) = C_P(T_4 - T_1) - T_o\ C_P \ln\frac{T_4}{T_1}$$

$$= 1.004\ [\ 738 - 290 - 298 \ln(\frac{738}{290})\ ] = 170.33 \text{ kJ/kg}$$

$$\eta_{II} = \frac{w_{net}}{\Phi_{qH} - \Phi_{qL}} = \frac{525.34}{695.56 - 170.33} = \mathbf{1}$$

Why is it 1?, the cycle is reversible so we could have said that right away.