Comparison Between Plastic Contact Hardness Models and Experiments

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COMPARISONS BETWEEN PLASTIC CONTACT HARDNESS MODELS
AND EXPERIMENTS

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ABSTRACT

The objective of this paper is to test empirical correlations available in the literature to predict the surface micro-hardness of metals. The surface micro-hardness is an important input parameter for thermal contact conductance models. The usual way of obtaining information about surface micro-hardness is by Vickers micro-hardness tests at various loads, which demands considerable time. The empirical correlations that are tested here need a single bulk hardness measurement at room temperature to estimate the micro-hardness variation near the surface at any temperature level. The application of the correlations is very easy and straightforward. Thermal contact conductance experimental data available in the literature for SS 304, Ni 200 and Zr-alloys are tested here. The results show that the empirical correlations worked very well for SS 304 and Ni 200. For Zr-4, the results were not satisfactory, indicating that this alloy responded to work-hardening in a different way from the other metals tested.

NOMENCLATURE

- $c_1$ Vickers correlation coefficient; MPa
- $c_2$ Vickers correlation coefficient
- $C_c$ dimensionless contact conductance (Eq. 1)
- $d$ Vickers average diagonal; µm

- $H$ Hardness; MPa
- $H^*$ Dimensionless hardness
- $h_c$ contact conductance; $W/m^2 \cdot K$
- $k$ solid thermal conductivity; $W/m \cdot K$
- $k_a$ harmonic mean thermal conductivity,
  \begin{align*}
  &= 2k_1k_2/(k_1 + k_2); \quad W/m \cdot K
  \end{align*}
- $m$ effective mean absolute asperity slope,
  \begin{align*}
  &= \sqrt{m_1^2 + m_2^2}
  \end{align*}
- $P$ apparent contact pressure; MPa
- RMS root-mean-square value
- $T$ temperature; $K$

Subscripts

- 0 reference
- 1, 2 surfaces 1 and 2 or solids 1 and 2
- $a$ apparent
- $c$ contact
- $r$ real
- $rm$ room
- $v$ Vickers

Greek Symbols

- $\sigma$ RMS surface roughness; $µm$
  \begin{align*}
  &= \sqrt{\sigma_1^2 + \sigma_2^2}; \quad µm
  \end{align*}

INTRODUCTION

Since actual surfaces present deviations from their idealized geometrical form, known as roughness and waviness, when two solids are put into contact, they will touch only at their highest asperities. The heat transfer across the interface between real solids is not as effective as if the solids were perfectly smooth and flat. Since the real contact area is much smaller than the apparent contact area ($< 2\%$), a resistance to heat flow, known as thermal contact
resistance, appears at the interface between the contacting solids. Heat transfer across the interface between two solids has been the subject of study of various researchers over many years. Contact heat transfer has many applications in engineering, such as ball bearings, microelectronic chips and nuclear energy.

When two solids are pressed together, the contacting asperities will deform and originate small spots of solid-solid contact. In the remaining portion of the apparent contact area the bodies are separated by a very thin gap. Heat transfer between two contacting solids can take place by three different modes: conduction through the contact spots, radiation through the gap in the remaining part of the apparent area and conduction through the gas that fills the gap. These heat transfer modes are treated separately and the sum of the conductances associated with each of these heat transfer modes is called the joint conductance. In this work, only the heat transfer associated with the solid to solid contact is considered. There are several models available in the open literature to predict the gap and the radiation conductance at the interface between contacting bodies.

Since the solid-to-solid contact heat transfer is more dominant than the gap and the radiation heat transfer, the heat flow from the hotter body must constrict in the region of the contact spots and then spread as it enters the colder body. This constriction and subsequent spreading of the heat flow results in an impediment to heat flow at the interface, referred to as contact resistance. The inverse of the contact resistance per unit apparent area is generally known as contact conductance.

There are several thermal contact conductance models available in the literature. Almost all the existing thermal contact conductance models are composed of three sub-models: thermal, geometrical and mechanical deformation models. The thermal model predicts the contact conductance for a given set of contact parameters: shape, size and number of contact spots. These contact parameters are obtained from a particular asperity mechanical deformation model, which can be elastic, plastic or elastoplastic. The deformation model requires a geometric model of the surface in order to be able to predict the contact parameters.

This work is focused on the issue of the deformation of the contacting asperities, i.e., the mechanical deformation model. Only surfaces undergoing plastic deformation are considered here. The crucial parameter that controls the plastic deformation of the contacting asperities is the surface micro-hardness. The usual way of obtaining this information is from Vickers micro-hardness tests at several indentation loads, which is a time consuming task. To avoid that, Sridhar and Yovanovich proposed empirical correlations to estimate the surface micro-hardness from the bulk hardness, which can be assessed through a single Brinell hardness test. It is also known that the hardness of metals is a function of temperature. Since hardness measurements are commonly made at temperatures different from the temperature encountered in thermal contact problems, Nho proposed correlations to correct the micro-hardness of metals for the actual interface temperature. The objective of this work is to test the accuracy of the correlations proposed by Sridhar and Yovanovich and by Nho to estimate surface micro-hardness. These correlations are used here to reduce thermal contact conductance experimental data available in the literature.

THEORY REVIEW

Cooper, Mikic and Yovanovich developed a theoretical thermal contact conductance model for contacting surfaces whose asperities experience plastic deformation. Various researchers in the thermal contact conductance field have employed this model during the last decades and it has been shown to be generally very accurate. Yovanovich correlated the model in a very simple form, as follows:

\[ C_c = \frac{h_c \sigma}{k_s m} = 1.25 \left( \frac{P}{H_c} \right)^{0.95} \]  

(1)

where \( h_c \) is the thermal contact conductance, \( k_s \) is the harmonic mean of the thermal conductivities of the contacting bodies, and \( \sigma \) and \( m \) are the RMS roughness and the mean absolute slope of the combined profile of the two contacting surfaces, respectively. The apparent contact pressure is \( P \) and \( H_c \) is the plastic contact hardness. This model is valid for isotropic surfaces, i.e., surfaces that do not present any directional roughness texture.

The plastic contact hardness \( H_c \), appearing in the expression above, is defined as the mechanical pressure that the contacting asperities can support. As the contacting surfaces are pressed against each other, the asperities of the harder surface indent the softer surface, which experiences plastic deformation. If the two materials have similar hardnesses, mutual deformation takes place. As a measure of \( H_c \), Cooper, Mikic and Yovanovich proposed the use of the bulk hardness of the softer of the two contacting materials. According to the authors, the bulk hardness should be obtained by indentation.
hardness tests, such as the Brinell test. However, Hegazy\textsuperscript{5} showed that the bulk hardness is not a good measure of the supporting contact pressure. According to the author, the use of bulk hardness makes the model of Cooper, Mikic and Yovanovich\textsuperscript{5} over predict experimental data by as much as 300\%, in some cases. This is because only the material very close to the surface suffers deformation under load, and the surface is generally much harder than the bulk of the material due to work hardening during the surface preparation. The bulk hardness tests employ relatively large indentation loads and therefore the indenter penetrates very deep into the surface. As a measure of $H_c$, Hegazy\textsuperscript{5} proposed the use of the surface micro-hardness instead of the bulk hardness. He proposed a model to predict the plastic contact hardness near the surface, which gave excellent agreement between thermal contact conductance theory and experiments. He measured the Vickers micro-hardness for various indentation loads. The Vickers micro-hardness test employs indentation loads as low as 0.1N, and the penetration is generally a few micrometers deep. Therefore, the hardness of the material very close to the surface is assessed. The obtained Vickers micro-hardness values ($H_v$) were then correlated to the respective diagonal length of the square indentations left by the indenter ($d_v$), which is proportional to the indentation depth, in the following form:

$$H_v = c_1 \left( \frac{d_v}{d_0} \right)^{c_2} \quad (2)$$

where $d_0$ is some arbitrary reference value, which is chosen, for convenience, to be $d_0 = 1\mu m$. The $c_1$ and $c_2$ coefficients appearing in the expression above are called the micro-hardness correlation coefficients, and they give a representation of the metal hardness variation with depth.

Song and Yovanovich\textsuperscript{6} proposed a model to predict the dimensionless contact pressure $P/H_c$, for a contacting pair with known $\sigma$, $m$, $c_1$ and $c_2$. The authors correlated their model in the following form:

$$\frac{P}{H_c} = \left[ \frac{P}{c_1(1.62\sigma/m)^{c_2}} \right] \frac{1}{1 + 0.071c_2} \quad (3)$$

The value obtained for $P/H_c$ from the expression above is directly used in Eq. (1) to predict the thermal contact conductance $h_c$.

So far, the only way of assessing the surface micro-hardness, i.e. assessing $c_1$ and $c_2$, is from Vickers micro-hardness tests at various loads, which demands considerable time. In order to overcome this problem, Sridhar and Yovanovich\textsuperscript{1} proposed empirical correlations to predict $c_1$ and $c_2$ as a function of the Brinell hardness, which can be obtained from a single indentation. The authors analyzed the surface micro-hardnesses of several metals and their relation to the bulk hardness. Using micro-hardness tests from Hegazy\textsuperscript{5} and Nho\textsuperscript{2} for SS 304, Ni 200 and Zr-alloys, as well as their own untreated and heat-treated tool steel and Ti-alloy specimens, Sridhar and Yovanovich\textsuperscript{1} proposed the following empirical correlations to estimate $c_1$ and $c_2$ based on the material bulk hardness:

$$\frac{c_1}{3178} = \left[ 4.0 - 5.77H_B^* + 4.0(H_B^*)^2 - 0.61(H_B^*)^3 \right] \quad (4)$$

and

$$c_2 = -0.370 + 0.442 \left( \frac{H_B}{c_1} \right) \quad (5)$$

where $H_B$ is the Brinell hardness and $H_B^* = H_B/3178$. These correlations are valid for metals with Brinell hardnesses between 1300 and 7600 MPa.

It is well known that the hardness of metals is a function of temperature: the higher the temperature, the softer the metal. Hardness measurements are generally conducted at room temperature, while the interface temperatures of actual thermal contact problems could reach much higher temperatures. Nho\textsuperscript{2} conducted experiments to analyze the effect of temperature on the micro-hardness correlation coefficients $c_1$ and $c_2$ of Al 6061-T5, SS 304 and Ni 200. The author found that $c_2$ is not sensitive to temperature variations between approximately 20 and 200\(^o\)C. On the other hand, $c_1$ decreased nearly exponentially with temperature. The author proposed the following correlations to estimate the $c_1$ coefficients for SS 304, Ni 200 and Al 6061-T5 at high temperatures:

For SS 304:

$$\frac{c_1}{c_1(T_{rm})} = \exp[-1.675 \times 10^{-3}(T - T_{rm})] \quad (6)$$

For Ni 200:

$$\frac{c_1}{c_1(T_{rm})} = \exp[-1.372 \times 10^{-3}(T - T_{rm})] \quad (7)$$

For Al 6061-T5:

$$\frac{c_1}{c_1(T_{rm})} = \exp[-1.19 \times 10^{-3}(T - T_{rm})] \quad (8)$$

which are valid for $20 < T < 200^{o}C$. In these equations, $c_1(T_{rm})$ is the value of $c_1$ obtained at room temperature $T_{rm}$.

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The objective of this work is to compare the models reviewed here with thermal contact conductance experimental data from Hegazy5 and Antonetti7 for conforming isotropic rough surfaces. The tests were performed under vacuum environment, with samples possessing various roughness levels, and for different metals. The data sets consist of the contact between bead-blasted/lapped surfaces. The study is focused on the analysis of the accuracy of the Sridhar and Yovanovich1 correlations for $c_1$ and $c_2$ (Eqs. 4 and 5). The accuracy of the correlations proposed by Nho2 to correct $c_1$ for the actual interface temperature is also analyzed in more detail in this paper.

**COMPARISON BETWEEN THEORY AND EXPERIMENTS**

Hegazy5 measured the thermal contact conductance of the interface between lapped/ bead-blasted surfaces of similar metals possessing various roughness levels. The author tested SS 304, Ni 200 and Zr-alloys. The first two are common materials employed by the industry and the Zr-alloys are generally used by the nuclear energy industry. The interface temperature during the tests were approximately 180°C.

Figure 1 shows the comparison between theory and SS 304 experimental data. Four pairs, presenting four different values of the ratio $\sigma/m$ are shown in this graph and are compared with the theoretical model (Eqs. 1 and 3). In this graph, $c_1$ and $c_2$ were obtained by the author from Vickers micro-hardness measurements at room temperature. The RMS difference for all 4 pairs is only 16%. The RMS difference of each pair can also be seen. The agreement is very good, in general, especially at higher contact pressures. The theory underpredicts experiments at light contact loads, which according to Milanez, Yovanovich and Culham3, is due to the truncation of the highest contacting asperities and is not related to the incorrect prediction of the supporting contact pressure $H_c$.

Since the interface temperature during the tests (approximately 180°C) were higher than the temperature during the Vickers micro-hardness tests, the data shown in Fig. 1 were reduced again using the $c_1$ coefficient corrected according to Eq. (6). The results are shown in Fig. 2. In this graph, the theory slightly underpredicts the experiments at high loads. As $c_1$ is corrected for temperature, the data points are displaced to the right in Fig. 2 because the plastic contact pressure ($H_c \sim c_1$) is smaller than in Fig. 1. However, the RMS difference is approximately the same (16%) in both cases.

![Figure 1 - SS 304 data from Hegazy5 ($c_1$ and $c_2$ measured, $c_1$ not corrected)](image)

In order to verify the accuracy of the correlations from Sridhar and Yovanovich1 for $c_1$ and $c_2$ (Eqs. 4 and 5), the same SS 304 data sets from Hegazy5 were reduced again. For SS 304 a bulk hardness value of $H_B = 1474\ MPa$ was reported by Hegazy5. For this value of bulk hardness, the Sridhar and Yovanovich1 correlations give $c_1(T_{rm}) = 6753\ MPa$ and $c_2 = -0.273$, against $c_1(T_{rm}) = 6271\ MPa$ and $c_2 = -0.229$, which were measured by Hegazy5. Figure 3 shows the comparison between theory and the SS 304 experimental data from Hegazy5 with $c_1$ and $c_2$ estimated using the correlations from Sridhar and Yovanovich1. Comparing Figs. 2 and 3, one can see that the results obtained using the estimated values of $c_1$ and $c_2$ are similar to the results using the measured values. Therefore the Sridhar and Yovanovich1 correlations were accurate in this case.

A behavior similar to the SS 304 data presented above was noticed with the Ni 200 data from Hegazy5 as well. Using $c_1$ obtained from measurements at room temperature leads the theory to predict experiments very well, with a RMS difference of 12%. Using the $c_1$ value corrected for the actual interface temperature, the theory slightly overpredicts the experiments at high loads, exactly in the same way as shown previously for SS 304. The RMS differences are 18% using the measured values of $c_1$ and $c_2$ and 15% using the predicted values using the Sridhar and Yovanovich1 correlations. Therefore, for the Ni 200 data set from Hegazy5, the Sridhar and
Yovanovich\textsuperscript{1} correlations were accurate, similar to the SS 304 data set.

The comparisons between Zr-2.5wt\%Nb and Zr-4 data and theoretical predictions for both the measured and the estimated values of $c_1$ and $c_2$ are presented in Figs. 4 and 5, respectively. The RMS difference of all data sets for the measured $c_1$ and $c_2$ values is 20\%. Using the values estimated by the Sridhar and Yovanovich\textsuperscript{1} correlations, the RMS difference is 50\%.

This reasonably large value of RMS implies that the correlations for $c_1$ and $c_1$ were not suitable for these data sets. The $c_1$ and $c_2$ values that Hegazy\textsuperscript{5} obtained from his specimens are quite different from the values Sridhar and Yovanovich\textsuperscript{1} used to obtain his correlations. Table 1 presents the $c_1$ and $c_2$ values measured by Hegazy\textsuperscript{5}, the values used by Sridhar and Yovanovich\textsuperscript{1} to obtain Eqs. (4) and (5), and the values that come from Eqs. (4) and (5). As can
be seen in this table, for Zr-2.5wt%Nb, SS 304, and Ni 200, the values computed using the Sridhar and Yovanovich\(^1\) correlations (last column) are in good agreement with the values measured by Hegazy\(^5\) (second column). Also for these metals, the values employed by Sridhar and Yovanovich\(^1\) (third column) to obtain Eqs (4) and (5) are the same values measured by Hegazy\(^5\). However, for Zr-4, Sridhar and Yovanovich\(^1\) used very different values to obtain their correlations (third column) from the values measured by Hegazy\(^5\) (second column). As a result, the values coming from the Sridhar and Yovanovich\(^1\) correlations (last column) are quite different from the measured values (second column). This explains why the comparison between theory and experimental data shown in Fig. 4 is less accurate than when compared with Fig. 5.

From these observations, it can be concluded that the Zr-4 alloy behaves differently than SS 304, Ni 200 and Zr-2.5 wt%Nb with respect to micro-hardness. The last three metals present consistent micro-hardness measurements in different samples, while it is clear from the second and third columns of Table 1 that the samples measured by Hegazy\(^5\) and the measurements used by Sridhar and Yovanovich\(^1\) to find the correlations given by Eq. (4) and (5) are quite different. The reason for this distinct behavior is probably related to the way different metals respond to the work-hardening during the surface preparation. However, it is still very difficult to predict how the work-hardening takes place during the surface preparation. The literature lacks works in this field.

Antonetti\(^8\) also measured the contact conductance between Ni 200 specimens presenting four distinct values of \(\sigma/m\). The data were reduced and plotted along with the isotropic contact conductance model (Eq. 1) and the results are shown in Figs. 6 to 8. Figure 6 was obtained using the measured values of \(c_1\) and \(c_2\), with \(c_1\) not corrected for temperature. Figure 7 was also obtained using the measured values of \(c_1\) and \(c_2\), but with \(c_1\) corrected for the actual interface temperature. Figure 8 was obtained with \(c_1\) and \(c_2\) predicted using Eqs. (4) and (5), and \(c_1\) corrected for temperature (Eq. 7). In Fig. 6 the agreement is excellent for the entire range of contact pressures, with a RMS difference of only 6%. When \(c_1\) is corrected for temperature (Fig. 7), the theory tends to slightly over predict the experimental data, with a RMS difference of 12%. The Sridhar and Yovanovich\(^1\) correlations for \(c_1\) and \(c_2\) make the theory predict the data points very well (Fig. 8), with a RMS difference of 8%. Therefore, the Ni 200 data from Antonetti\(^8\) presented a behaviour very similar to the Ni 200 and SS 304 data from Hegazy\(^5\).

### Table 1 - Bulk Hardness (\(H_B\) [MPa]), \(c_1\) [MPa] and \(c_2\)

<table>
<thead>
<tr>
<th>Material</th>
<th>Measured by Hegazy(^5)</th>
<th>Used to obtain Eqs. (4) and (5) (Sridhar and Yovanovich(^1))</th>
<th>Computed using Eqs. (4) and (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zr-2.5wt%Nb</td>
<td>(c_1 = 5884)</td>
<td>(c_1 = 5884)</td>
<td>(c_1 = 6190)</td>
</tr>
<tr>
<td>(H_B = 1727)</td>
<td>(c_2 = -0.267)</td>
<td>(c_2 = -0.267)</td>
<td>(c_2 = -0.237)</td>
</tr>
<tr>
<td>Zr-4</td>
<td>(c_1 = 3239)</td>
<td>(c_1 = 5677)</td>
<td>(c_1 = 6372)</td>
</tr>
<tr>
<td>(H_B = 1913)</td>
<td>(c_2 = -0.145)</td>
<td>(c_2 = -0.278)</td>
<td>(c_2 = -0.249)</td>
</tr>
<tr>
<td>Ni 200</td>
<td>(c_1 = 6304)</td>
<td>(c_1 = 6304)</td>
<td>(c_1 = 6309)</td>
</tr>
<tr>
<td>(H_B = 1668)</td>
<td>(c_2 = -0.264)</td>
<td>(c_2 = -0.264)</td>
<td>(c_2 = -0.245)</td>
</tr>
<tr>
<td>SS 304</td>
<td>(c_1 = 6271)</td>
<td>(c_1 = 6271)</td>
<td>(c_1 = 6753)</td>
</tr>
<tr>
<td>(H_B = 1472)</td>
<td>(c_2 = -0.229)</td>
<td>(c_2 = -0.229)</td>
<td>(c_2 = -0.273)</td>
</tr>
</tbody>
</table>

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perceiving plastic deformation. The surface micro-

hardness coefficients $c_1$ and $c_2$ used here, which are 

very important input parameters to the thermal con-

tact conductance model, were both the measured 

values and the values estimated using the empirical 

correlations proposed by Sridhar and Yovanovich\(^1\).

The correlations proposed by Nho\(^2\) to correct the 
$c_1$ coefficient for the actual temperature of the inter-

face were also tested here. The correlations proposed 

by Nho\(^2\) to correct $c_1$ for the actual interface tem-

perature in general make the theory slightly overpre-

dict the experimental data. The experimental data 

used here appears to agree better with the predic-

tions when the $c_1$ values at room temperature are 

used instead of the corrected values.

For Ni 200 and SS 304, the $c_1$ and $c_2$ values com-

puted using the Sridhar and Yovanovich\(^1\) correla-

tions gave very good results. The RMS differences 

for each data set vary only a few percent if one uses 

the estimated $c_1$ and $c_2$ instead of the measured val-

ues. For the Zr-alloys, the results were not very 

good, especially for Zr-4. It is believed that this alloy 

responds to work-hardening during surface prepara-

tion in a different way to the other metals tested.

Further studies are needed in order to test the va-

lidity of the empirical correlations for $c_1$ and $c_2$ over 

a larger spectrum of metals.

**SUMMARY AND CONCLUSIONS**

The objective of this work was to compare ther-

mal contact conductance experimental data between 

conforming rough surfaces under vacuum against 

models available in the open literature. The avail-

able data include SS 304, Ni 200, Zr-4 and Zr-

2.5wt\%Nb pairs from Hegazy\(^4\) and Antonetti\(^7\). The 

contact conductance models analyzed here are based 

on the model of Cooper et al.\(^3\), for asperities ex-

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