Influence of Discrete Heat Source Location on Natural Convection Heat Transfer in a Vertical Square Enclosure

A numerical study is carried out to investigate the influence of discrete heat sources on natural convection heat transfer in a square enclosure filled with air. The enclosure has two vertical boundaries of height \( H \); one of them is cooled at \( T_c \), and the other has discrete heat sources \([\text{isoflux} (q = c) \text{ or isothermal} (T_b = c)]\). The enclosure has two horizontal adiabatic boundaries of length \( L \). Results are reported for \( 0 \leq Ra \leq 10^6 \), \( Pr = 0.72 \), \( A = 1 \), aspect ratio \( \varepsilon \), the relative size of the heat source to the total height, lies in the range \( 0.25 \leq \varepsilon \leq 1 \) and the discrete heat sources are located at the top or the bottom of the enclosure. Verification of numerical results is obtained at \( Ra = 0 \) (conduction limit) with analytical conduction solutions. In addition, a comparison with experimental and numerical data is made which also shows good agreement. The relationships between both \( Nu, \Delta Nu \) (change of thermal conductance) and \( Ra \) based on scale length (the size of the heat source \( S \) divided by the aspect ratio \( A \)) are also investigated here. A relationship \( Nu \) and \( Ra \), based on scale length obtained from analytical solutions is correlated as \( Nu = Nu (Ra, \varepsilon) \). In addition, extrapolation correlations of \( Nu \) over the very high range of Rayleigh numbers \( (Ra \geq 10^7) \) are developed.

1 Introduction

The influence of discrete heat source size and location and boundary conditions on natural convection heat transfer within a square enclosure filled with air, is investigated in this work using a numerical finite difference technique.

The enclosure, as shown in Fig. 1, consists of two vertical boundaries of height \( H \), and two horizontal boundaries of length \( L \). One vertical boundary is maintained at \( T_c \) and the other has a discrete heat source \([\text{isoflux} q \text{ or isothermal} T_b]\) on an otherwise adiabatic surface. The top and bottom horizontal boundaries are adiabatic.

There are few investigations which have examined the effect of location of the discrete heat source inside the square cavity. Chu and Churchill [1] were the first to study this problem numerically. They examined the effect of the location in the range of \( Ra_H \) from 0 to \( 10^8 \). Chu and Churchill [1] found that \( Nu \) was proportional to \( Ra_H \) for any location of the discrete heat source. Flack and Turner [2] and Turner and Flack [3] also studied the previous effect experimentally but at high Rayleigh number (based on the cavity height) for \( 4.3 \times 10^6 \leq Ra_H \leq 6.48 \times 10^6 \) and confirmed the observations of Chu and Churchill [1]. Recently, Cesini et al. [4] studied the effect of the location of the discrete heat source numerically and experimentally with the discrete heat source at the bottom or center of the wall and reported the same observations as the previous studies [1–3]. However, the difference between Cesini et al. [4] and Chu and Churchill [1] at \( A = 1, \varepsilon = 0.5, P/H = 0.5 \) and \( Ra_H = 2.5 \times 10^8 \), was approximately 5 percent and between Cesini et al. [4] and Turner and Flack [3] at \( A = 1, \varepsilon = 0.5, P/H = 0.5 \) and \( Ra_H = 3 \times 10^9 \), was approximately 18 percent. These comparisons are based on the results of Cesini et al. [4].

The objectives of this study are to examine the effect of the location of the discrete heat source on the rate of heat transfer in the range of \( Ra \) (based on \( S/A \)) from 0 to \( 10^6 \), and also to examine the effect of boundary conditions of the discrete heat.
source (IFDHS or ITDHS). Finally, examination of the scale length \( S/A \) is made, which was developed analytically by Refai and Yovanovich [5] in order to develop design correlations for this problem. Design correlations in microelectronics applications are required to cover the high range of the Rayleigh number.

This paper is organized as follows. In the following section, the governing equations are stated with proper assumptions. In the third section, the definition of the Nusselt number is given. In section 4, the numerical technique is presented and the numerical results are discussed. In addition, the obtained and correlated results are discussed in section 4. Finally, conclusion is given in section 5.

2 The Governing Equations

The flow is assumed to be laminar, two-dimensional, and incompressible with constant density except in the buoyancy term of the momentum equation (the Boussinesq approximation). The nondimensionalized governing equations are:

\[
\begin{align*}
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} &= 0 \\
\frac{DU}{D\tau} &= -\frac{\partial P}{\partial X} + Pr \nu^2 U \\
\frac{DV}{D\tau} &= -\frac{\partial P}{\partial Y} + Pr \nu^2 V + RaPr \theta \\
\frac{D\theta}{D\tau} &= \frac{v^2 \theta}{\kappa}
\end{align*}
\]

with the nondimensional variables defined as:

\[
\begin{align*}
U &= \frac{u(S/A)}{\alpha} \\
V &= \frac{v(S/A)}{\alpha} \\
X &= \frac{x}{(S/A)} \\
Y &= \frac{y}{(S/A)} \\
P_d &= \frac{P_d(S/A)^2}{\rho \alpha^2 A^2} \\
\theta &= \frac{T-T_c}{T_h-T_c} \text{ IFDHS}
\end{align*}
\]

3 Definition of the Nusselt Number

The local and average thermal energy balances at \( x = L \) give the local heat transfer coefficient:

\[
h \frac{\partial \theta}{\partial x} = \frac{k}{\rho c_p} \int_{y_p}^{y_s} \frac{\partial \theta}{\partial y} dy
\]

and the average value:

\[
h = \frac{k}{\rho c_p} \int_{y_p}^{y_s} \frac{\partial \theta}{\partial y} dy
\]

Nomenclature

- \( A \): aspect ratio of cavity, \( A = H/L \)
- \( A_s \): surface area per unit length, m
- \( C_p \): specific heat at constant pressure, kJ/kg\( \cdot \)K
- \( g \): gravitational acceleration, m/s\(^2\)
- \( H \): height of the cavity, m
- \( h \): total coefficient of heat transfer, W/m\( ^2 \)K
- \( h_{\text{cond}} \): coefficient of heat transfer by conduction, Eq. (19), W/m\( ^2 \)K
- \( h_{\text{conv}} \): coefficient of heat transfer by convection, Eq. (19), W/m\( ^2 \)K
- \( h_p \): local coefficient of heat transfer, W/m\( ^2 \)K
- \( k \): thermal conductivity, W/m\( \cdot \)K
- \( L \): width of cavity, m
- \( Nu \): average Nusselt number, \( Nu = h(S/A)/k \)
- \( Nu_t \): average Nusselt number, \( Nu = hS/k \)
- \( P \): distance from the bottom of the enclosure, m
- \( P_d \): non-dimensional dynamic pressure
- \( P_{d} \): dynamic pressure, N/m\(^2\)
- \( Pr \): Prandtl number, \( Pr = v/\alpha \)
- \( Q \): total heat flow rate per unit length, W/m
- \( Q_{\text{cond}} \): conduction heat flow rate per unit length, W/m
- \( Q_{\text{conv}} \): convection heat flow rate per unit length, W/m
- \( Q_{\text{loss}} \): heat loss per unit length, W/m
- \( Q_{\text{out}} \): radiation heat flow rate per unit length, W/m
- \( q \): heat flux, W/m\(^2\)
- \( Ra \): Rayleigh number, \( Ra = (S/A)^3 \beta g (T_h - T_c)/\alpha \nu \)
- \( Ra_{\text{it}} \): Rayleigh number, \( Ra = H^3 \beta g (T_h - T_c)/\alpha \nu \)
- \( R_m \): nondimensional material resistance, \( R_m = r_{m} / (k \cdot A \cdot (\text{unit length})) \)
- \( r_m \): material resistance, \( r_m = 1/(k \cdot A \cdot (\text{unit length})) \)
- \( R_t \): nondimensional total resistance, \( R_t = \bar{\theta}/k \cdot q \cdot (S/A) \)
- \( S \): length of discrete heat source, m
- \( T \): temperature, K
- \( T_m \): mean temperature, \( T_m = (T_h + T_c)/2 \), K
- \( t \): time, s
- \( U \): dimensional velocity component in X direction, m/s
- \( u \): dimensional velocity component in x direction, m/s
The total heat flow rate at \( x = L \), is therefore,

\[
Q = q \cdot S = k \int_{0}^{L} \left( \frac{\partial \theta}{\partial y} \right) dy = h \cdot S \cdot \theta_c
\]  

(7)

The area-average Nusselt number can be defined as:

\[
\text{Nu} = \frac{h(S/A)}{k} = \frac{q(S/A)}{\theta_c}
\]  

(8)

which is based on the scale length \( S/A \).

4 Results and Discussion

A numerical finite difference technique, the Marker and Cell method MAC, developed by Harlow and Welch [6], is modified to solve the governing Eqs. (1-4). This method uses a different formulation with primitive variables as the dependent variables. More details about this technique can be found in [5, 7]. In the present study, the air layer was divided into \((24 \times 24)\) cells which are surrounded by a single layer of boundary cells marking the computational matrix \((26 \times 26)\). The steady-state solution was obtained as the limit of the transient calculations using a nondimensional time step, \( \Delta \tau \) as low as \( 8 \times 10^{-5} \). This gave an average computational time per cell per time step of about 0.002 s on a PC AT 386 (A = 1, \( Ra = 10^6 \), \( \varepsilon = 1 \), IFDHS). Also by increasing the number of cells up to \( 38 \times 38 \), a slight difference (approximately 0.9 percent) between the average Nusselt number was found.

Before studying the effect of the different parameters on the average Nusselt number, it was necessary to examine the accuracy of the MAC technique; this test was reported in more detail by Refai and Yovanovich [5]. This study contains only the comparison between the numerical solution and the analytical results when the discrete heat source is located at the top or the bottom of the enclosure as shown in Tables 1 and 2. The total resistance \( R_{\text{t}} \) consists of the material resistance \( R_{\text{m}} \) and the convection resistance \( R_{\text{c}} \). The material resistance depends upon the geometry of the enclosure, while the convection resistance is a function of the size and the location of the discrete heat source as shown in Tables 1 and 2.

Remark 1
There is very good agreement within 1 percent, between the...
numerical results and the analytical solutions for $R_e$ at $R_e = 0$.

Figures 2 and 3 show the relationships between the Nusselt number and the Rayleigh number for IFDHS and ITDHS, respectively. The location of the discrete heat source is varied between the bottom and top of the right vertical boundary. The trend of the relationships between $Nu$ and $Ra^*$ or $Ra$ as shown in Figs. 2(a) and 3(a), when the discrete heat source is at the bottom of the wall, is the same as the trend when the discrete heat source is at the center of the wall, which was studied by Refai and Yovanovich [5]. It is found that when $Ra^*$ or $Ra$ is greater than 300, $Nu$ decreases with increasing $\epsilon$. In contrast, when the discrete heat source is at the top as shown in Figs. 2(b) and 3(b), $Nu$ decreases with increasing Rayleigh number up to a certain value (approximately at Rayleigh equal 300). Chu and Churchill [1] found that $Nu$ increases at any location of the discrete heat source with $Ra_H$. However, a cautionary remark, Yaghoubi and Incopera [8] state that the coarse mesh of Chu and Churchill [1] can lead to spurious flows, even with computational stability.

In addition, ElSherbiny [9] determined experimentally the effect of location (full contact heat source $\epsilon = 1$) using three heaters on the hot wall. He measured $Nu$ for each heater; $Nu$ for the upper heater was found to decrease up to a certain $Ra$ and then increase again, as shown in Fig. 4. Figure 4 also shows the comparison between the present results and ElSherbiny [9]. The trend of the relationships between the Nusselt number and the Rayleigh number is the same, and there is good agreement between [9] and this study.

Table 3 shows the comparison between the present results and the correlation of the numerical results of Cesini et al. [4] and their experimental work was 17 percent (their numerical results are higher than their experimental data). They suggest that this difference is probably due to the difficulties in interpreting the interferograms near the bottom corner of the enclosure, due to the high concentration of fringes. Furthermore, the effect of thermal conduction of the insulating walls should be taken into consideration.

Refai and Yovanovich [5] also showed very good agreement between their numerical results and the experimental results of MacGregor and Emery [10], and Eckert and Carlson [10] when $\epsilon = 1$.

Figure 5 shows the variation of the air temperature profiles at the first column of cells away from the smallest discrete heat source ($X = 0.98$, $\epsilon = 0.25$) for $Ra = 625$. The trend of the temperature profiles is identical at any location of the discrete heat source. It usually starts at a low temperature and increases. However, the average temperatures are 0.836, 0.891 and 0.96 (bottom, center, top). The increase in the average temperature with increase in elevation implies there is a decrease in the heat transfer coefficient or average Nusselt number. Figure 6 also shows the variation of the temperature profiles.
at the first column of cells away from the smallest discrete heat source ($\epsilon = 0.25$) when it is located at the top for various Rayleigh number ($0 \leq Ra \leq 625$) and ITDHS. The trend of the temperature profiles is the same as in Fig. 5. However, the temperature increases with increasing Rayleigh number ($0 \leq Ra \leq 625$). After this the distribution of the temperature at the bottom of the discrete heat source decreases with increasing $Ra$. Therefore $Nu$ decreases to $Ra = 62.5$ and then decreases when the cooling layer begins moving to the top of the enclosure.

The relationship between $\Delta Nu$ and the Rayleigh number was developed by Refai and Yovanovich [5] (see Appendix), to assist in the development of correlations of $Nu = Nu (Ra, \epsilon)$ or $Nu = Nu (Ra, \epsilon)$ and to make it possible to extrapolate to very large $Ra$. The change in Nusselt, $\Delta Nu$, due to fluid motion is defined by the following equation:

$$\Delta Nu + Nu(Ra = 0) = Nu(Ra)$$

(9)

Figures 7 and 8 show this relationship for both IFDHS and ITDHS when the discrete heat source is located at the top and then the bottom of the enclosure. $\Delta Nu$ increases with increasing Rayleigh and all curves of $\Delta Nu$ approach a common asymptote (approximately at $Ra = 10^6$). This asymptote is the curve of the full contact heat source $\epsilon = 1$. (Refai and Yovanovich [5] also obtained this asymptote when the discrete heat source was located at the center of the enclosure). Therefore there is negligible effect of the size of the discrete heat source for $Ra > 10^6$.

**Remark 2**

The relationships between $\Delta Nu$ and $Ra$ and $Ra^*$ approach a common asymptote. This asymptote is the same whether the discrete heat source is located at the top, center [5] or bottom. Therefore, there is negligible effect of the location or the size of the discrete heat source at high Rayleigh number ($Ra > 10^6$).

It is very important to correlate numerical data, since it is difficult, if not impossible, to use the results otherwise. If the design equation is general, it will be more useful in engineering applications. In microelectronics applications, correlations are required to cover the high range of the Rayleigh number. Refai and Yovanovich [5] developed two equations to cover the range of $0 \leq Ra \leq 10^6$, when the discrete heat source (IFDHS) or (ITDHS) is at the center by using the blending method of Churchill and Usagi [12]. These equations are also applicable when the discrete heat source is located at the bottom by slight modifications as follows:

1. IFDHS, $0 \leq Ra^* \leq 10^6 \epsilon^4$

$$Nu = \left[ \epsilon^{0.11} \left( e^{-0.289} \right)^m + \left( 0.21 e^{-0.289} Ra^* 0.221 \left( 1.262(0.7)^{12} - \epsilon^{-0.12} \right)^{12} \right)^2 \right]$$

(10)

2. ITDHS, $0 \leq Ra \leq 10^6 \epsilon^3$

$$Nu = \left[ \epsilon^{1.7} e^{1.105 \epsilon^4 (0.146 e^{-0.259} Ra^{0.287} (1.25 \epsilon^{-0.033})^{12})^{12}} \right]^{12}$$

(11)

where $m$ has two values:

(i) $m = 0$ when the discrete heat source is at the center or $\epsilon = 1$
(ii) $m = 1$ when the discrete heat source is at the bottom and $n$ also has two values only when the discrete heat source is at the bottom:

(i) $n = 0$ for $0.5 \leq \epsilon \leq 1$
(ii) $n = 1$ for $\epsilon = 0.25$.

The maximum difference of 10 percent between the correlation equations and the numerical results occurs when $Nu$ lies near the intersection of the conduction and laminar regime asymptotes. The average difference is 4.7 percent and the standard deviation is 2.1 percent. However, when the discrete heat source is located at the top, it is difficult to correlate the relationships between $Nu$ and $Ra$, because $Nu$ first decreases and then increases with increasing $Ra$. Figures 7 and 8 show that the relationships between $\Delta Nu$ and Rayleigh number go to one asymptote for $\epsilon = 1$ at high Rayleigh number ($Ra \geq 10^6$). Therefore the extrapolation equations for IFDHS and

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Table 4  Comparison between present study and previous numerical and experimental studies
at $A = 1$ and $e = 1$ (ITDHS)

<table>
<thead>
<tr>
<th>$\text{Ra}$</th>
<th>$\text{Eq.(13)}$</th>
<th>$\text{Nobile}_{\text{num}}$</th>
<th>$\text{Markatos}_{\text{num}}$</th>
<th>$\text{Abrams}_{\text{num}}$</th>
<th>$\text{Catton}_{\text{exp}}$</th>
<th>$\text{Cowan}_{\text{exp}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^7$</td>
<td>16.7</td>
<td>16.9</td>
<td>10.8</td>
<td>32.05</td>
<td>1.6</td>
<td>3.27</td>
</tr>
<tr>
<td>$10^8$</td>
<td>28.6</td>
<td>25.5</td>
<td>10.8</td>
<td>35.6</td>
<td>1.6</td>
<td>3.27</td>
</tr>
<tr>
<td>$10^9$</td>
<td>36.2</td>
<td>30.2</td>
<td>16.5</td>
<td>55.1</td>
<td>2.3</td>
<td>3.27</td>
</tr>
<tr>
<td>$10^10$</td>
<td>79.8</td>
<td>54.7</td>
<td>31.5</td>
<td>67.3</td>
<td>15.7</td>
<td>3.27</td>
</tr>
<tr>
<td>$10^11$</td>
<td>139.4</td>
<td>67.2</td>
<td>37.4</td>
<td>107.5</td>
<td>22.9</td>
<td>3.27</td>
</tr>
</tbody>
</table>

1 $\Delta Nu = 0.18(Ra + 10^4)^{0.14}m + 0.0227(Ra)^{0.37}$
2 $\Delta Nu = 0.0725Ra^{0.35}$ where $(Pr = 0.729)$
3 $\Delta Nu_{\text{exp}} = [(Nu_{\text{exp.(13)}} - Nu_{\text{num}})/Nu_{\text{exp.(13)}}] \times 100\%$

ITDHS of Refai and Yovanovich [5] are valid here with some modifications in the conduction and laminar limits. The correlations are listed below for discrete heat sources at top, center or bottom, for $Ra \leq 10^6$. For $10^6 \leq Ra \leq 10^9$, the isoflux correlation equation is

$$\text{Nu} = 0.18(Ra + 10^4)^{0.14}m + 0.0227(Ra)^{0.37}$$

(12)

For $10^7 \leq Ra$, the isothermal correlation equation is

$$\text{Nu} = 0.0725Ra^{0.35}$$

(13)

where $m$ has two values:
(i) $m = 0$ when the discrete heat source is at the center or bottom
(ii) $m = 1$ when the discrete heat source is at the top.

There is no use of Eqs. (12) and (13) when $[10^6 \leq Ra \leq 10^9]$ at $e < 1$, where the difference is 34 percent for $e = 0.5$, $P/H = 0.25$ at $Ra = 10^6$. However, one can use Figs. 7 and 8 in this range and then add the effect of conduction from Tables 1 or 2 ($Nu(Ra = 0) = 1/Ra$). For example, at $e = 0.5$, $Ra = 10^5$, ITDHS and $P/H = 0.25$, $\Delta Nu$ from Fig. 8(a) is 11.2 and $Nu$ is 12.

Table 4 shows the comparison between Eq. (13), the present study, and the previous numerical and experimental studies [13–17]. This comparison shows that there is reasonable agreement in the range of Rayleigh number $10^7 \leq Ra \leq 10^9$ between the previous studies [13–17] and the present investigation. On the other hand, in the Rayleigh number range of $10^8 < Ra < 10^9$, the difference between the present results and the previous studies lies between 8.4 percent and 40.8 percent. Table 4 shows the Eq. (13) gives good predictions of Nusselt number over the range of Rayleigh number $10^7 \leq Ra \leq 10^9$; however, the physical interpretation is unknown.

Remark 3

The correlation equations, Eqs. (12) and (13), are in very good agreement with the results of previous experimental studies of MacGregor and Emery [10] and Eckert and Carlson [11] as shown in the study of Refai and Yovanovich [5], where they developed extrapolation equations for discrete heat sources at the center, i.e., without [15].

5 Conclusions

The relationship between $\text{Nu}$ and $\text{Ra}$ has the same trend whether the discrete heat source is located at the center or bottom, but, when the discrete heat source is at the top, $\text{Nu}$ decreases up to $Ra = 300$ and then increases. There is agreement between this observation when the discrete heat source is at the top and the investigation of Eilsherby [9].

In addition, there is agreement between the experimental and numerical work of Cesini et al. [4] and the present study. In addition, Table 4 also shows the agreement between the present study and the previous numerical and experimental studies [13–17] at high Rayleigh number; however, the physical interpretation is unknown.

References

13 Nobile, E., Sousa, A. C. M., and Barozzi, G. S., "Turbulent Buoyant
From the Fourier equation
\[
Q_{\text{cond}} = -k(T_{x=0}) \cdot A_s \cdot (\Delta T/\Delta x)_{x=0}
\]
(16)

At \(Ra = 0\),
\[
Q_{\text{cond}} = k(T_m) \cdot A_s \cdot (T_h - T_c)/L
\]
(17)

\[
h_{\text{cond}} = h_{\text{cond}} \cdot A_s \cdot (T_h - T_c)
\]
(18)

Therefore, \(Q_{\text{cond}} = k(T_m)/L\) where \(L = (S/A)\) at \(\epsilon = 1\).

We can rewrite Eq. (14) in the following form:
\[
h \cdot A_s \cdot (T_h - T_c) = h_{\text{cond}} \cdot A_s \cdot (T_h - T_c) + h_{\text{conv}} \cdot A_s \cdot (T_h - T_c)
\]
(19)

Multiply Eq. (19) by \((S/A)/(k(T_m) \cdot A_s \cdot (T_h - T_c))\) in order to nondimensionalize.

The equation becomes:
\[
h \cdot (S/A)/k(T_m) = [h_{\text{cond}} \cdot (S/A)]/k(T_m)
\]
\[
+ [h_{\text{conv}} \cdot (S/A)]/k(T_m)
\]
(20)

or as
\[
Nu = h_{\text{cond}} \cdot (S/A) + h_{\text{conv}} \cdot (S/A)
\]
(21)

Recognizing that \(Nu_{\text{cond}} = Nu(Ra = 0)\) and defining \(\Delta Nu = Nu_{\text{cond}}\) we conclude that
\[
\Delta Nu + Nu(Ra = 0) = Nu(Ra)
\]
(22)