AN INVESTIGATION OF THE RELATIONSHIP BETWEEN PACKAGING DENSITY AND EFFECTIVE THERMAL CONDUCTIVITY IN LAMINATED PRINTED CIRCUIT BOARDS

by

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ABSTRACT

An experimental and numerical investigation of the thermal behavior of fibreglass epoxy printed circuit boards with multiple copper layers and discrete, surface mounted heat sources in a vertical orientation with natural convection cooling is presented. Through a comparison of measured and numerical values for three different heat source configurations, the effect of component density and power on the effective thermal conductivity for a single layer model is determined. Previous analytical work has suggested that this effective thermal conductivity is bounded by lower and upper limits defined using the series and parallel conduction paths, respectively. However, the following investigation shows that when the fluid side resistance controls the heat transfer, as in natural convection cooling, the effective thermal conductivity required by various numerical and analytical solutions exceeds the previously defined upper limit and an alternate model is required.

NOMENCLATURE

\( a, b \) - major, minor axis lengths, elliptical area of influence, m
\( h \) - convective heat transfer coefficient, \( W/m^2K \)
\( k \) - thermal conductivity, \( W/mK \)
\( k_{eff} \) - effective thermal conductivity, \( W/mK \)
\( L, W \) - board dimensions, m
\( L, A \) - generalized length, area for resistance calculation, m, \( m^2 \)
\( N \) - number of circuit board layers
\( r_{eff} \) - effective radius of influence, m
\( R \) - thermal resistance, \( K/W \)
\( t \) - board material layer thickness, m

Subscripts
\( f \) - fluid-side quantity
\( i \) - board layer
\( p \) - based on the parallel resistance path
\( s \) - based on the series resistance path
INTRODUCTION

The thermal characterization of multilayered printed circuit boards is an essential element in the design and testing of microelectronics equipment. Efficient and accurate modeling techniques allow designers to optimize product reliability by judiciously selecting and placing component packages, thereby limiting thermally induced failures. This thermally-driven design process is vital in modern telecommunications and computer applications, where heat conduction in the circuit board is often used to augment the natural convection cooling of high-powered, high heat flux components.

Although the open literature includes investigations of both heat conduction within composite materials and heat distribution in homogeneous base plates with varying source strength and placement, there are currently no models available for the thermal characterization of multilayered printed circuit boards (PCBs) that reflect the influence of varying discrete heat source configurations. The only method available for the design engineer to model the full multilayer heat conduction problem has been through experiments or numerical simulations, such as those available using CFD packages like FLOOTHERM (1995).

The thermal modeling of multilayered printed circuit boards is a challenging problem due to the varied geometries and the range of thermal properties typically found in most modern microelectronics circuitry. The common laminates found in typical PCBs are 1 ounce (0.035 mm) copper tracking and ground planes and epoxy-based, electrically-insulating layers with thicknesses up to 1.5 mm, depending on the number of layers in the board. Given these materials and dimensions, thermal conductivity and thickness ratios between the PCB components may be in excess of 1000:1 and 500:1, respectively, and the overall aspect ratio between the the planar dimensions of the PCB and its thickness can be as large as 10000:1. Thermal modeling of this complex composite body using conventional numerical techniques requires a highly discretized solution domain to attain more reasonable aspect ratios for each control volume. As a result, numerical simulations of multilayered printed circuit boards often involve excessive numbers of control volumes, long solution times, poor convergence characteristics, and questionable accuracy.

One method that is often used to model this complex heat transfer problem involves lumping the conductive properties of the individual layers into a single, effective thermal conductivity, $k_{\text{eff}}$. This allows the multilayered PCB to be modeled as a single, homogeneous material, greatly simplifying any subsequent thermal analyses. When applied to currently available numerical or analytical solutions, such as FLOOTHERM (1995) or META (Culham et al., 1991), the result is a more efficient model that converges to a solution more quickly than its multilayered counterpart.

In the following investigation, experimental methods are used to determine component temperatures for a natural convection cooled, vertically oriented, three layer PCB with one, three, and five surface mounted sources. FLOOTHERM finite volume models of these same configurations with a single layer PCB are then used to determine the values of $k_{\text{eff}}$ that best match the experimental and numerical component temperatures. Preliminary testing indicates that the effective conductivity values required for accurate predictions by a single layer board model such as FLOOTHERM are significantly larger than the values typically recommended by most handbooks or software user’s manuals. This behavior seems to suggest that the effective thermal conductivity for certain multilayer arrangements and test conditions may not be bounded by the lower and upper limits proposed in previous work (Lemczyk et al., 1992).

The objectives of this paper are to present observations concerning the behavior of numerical and analytical solutions for single layer PCBs as a function of the effective conductivity of the board, and to
propose a relationship between $k_{eff}$ and the package density for the specific case of vertically oriented, natural convection cooled PCBs with various source configurations.

**THEORETICAL ANALYSIS**

The classical heat conduction analysis for a composite system of materials, such as multilayered PCBs, involves the definition of two limiting cases which describe the upper and lower bounds on the thermal resistance. These bounds can be defined for this particular case using the equivalent thermal resistance networks presented in Fig. 1. The following is a definition for the thermal resistance due to conduction through a homogeneous material of constant cross sectional area:

$$R = \frac{L}{kA}$$ (1)

where $L$ and $A$ are the length and cross sectional area of the conduction path and $k$ is the thermal conductivity. The lower and upper bounds corresponding to the series and parallel resistive paths, Fig. 1(a) and (b), can be determined by:

$$R_s = \sum_{i=1}^{N} \frac{t_i}{k_i LW}$$ (2)

$$R_p = \left(\sum_{i=1}^{N} \frac{k_i t_i W}{L/2}\right)^{-1}$$ (3)

where $L$ and $W$ are the length and width of the board and $t_i$ and $k_i$ are the thickness and conductivity of each of the layers.

Most calculations of effective thermal conductivity in laminated substrates are based on a weighted average of the series and parallel heat flow paths. For surface mounted heat sources, the series path is directly through the board and the thermal conductivity of the least conductive material dominates, acting as the controlling resistance between the source and sink. Unlike the series path, the parallel path is not controlled by any single layer because several alternate paths are provided for heat flow. As a result, the mean conductivity along the parallel path is often many times larger than the conductivity along the series path.
The previous work by Lemczyk et al. (1992) suggests the following formulations for the lower and upper bounds on the thermal conductivity for a multilayered PCB:

\[
k_s = \frac{\sum_{i=1}^{N} t_i}{\sum_{i=1}^{N} (t_i/k_i)} \tag{4}
\]

\[
k_p = \frac{\sum_{i=1}^{N} (k_i t_i)}{\sum_{i=1}^{N} t_i} \tag{5}
\]

While the actual heat flow path from the source through the board is not known, an average of these limiting values. Eqs. (4) and (5), is often used as a good estimate for \( k_{eff} \). Lemczyk et al. (1992) suggest that the harmonic mean of the series and parallel limits provides the best estimate of an effective conductivity for a multilayered PCB.

Table 1 presents the physical dimensions and properties of the multilayered printed circuit board used in this investigation, as well as the suggested values for the upper and lower bounds on the thermal resistance and the effective conductivity.

<table>
<thead>
<tr>
<th>layer</th>
<th>( t ) (mm)</th>
<th>( L ) (mm)</th>
<th>( W ) (mm)</th>
<th>( k ) (W/mK)</th>
<th>( t/(k L W) ) ((K/W))</th>
<th>( L/(2 k t W) ) ((K/W))</th>
</tr>
</thead>
<tbody>
<tr>
<td>cu1</td>
<td>0.036</td>
<td>0.15</td>
<td>0.15</td>
<td>386.0</td>
<td>4.15\times10^{-6}</td>
<td>35.98</td>
</tr>
<tr>
<td>FR4</td>
<td>1.575</td>
<td>0.15</td>
<td>0.15</td>
<td>0.41</td>
<td>0.171</td>
<td>774.3</td>
</tr>
<tr>
<td>cu2</td>
<td>0.036</td>
<td>0.15</td>
<td>0.15</td>
<td>386.0</td>
<td>4.15\times10^{-6}</td>
<td>35.98</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>OVERALL THERMAL RESISTANCE ((K/W))</th>
</tr>
</thead>
<tbody>
<tr>
<td>series path</td>
</tr>
<tr>
<td>parallel path</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>EFFECTIVE THERMAL CONDUCTIVITY ((W/mK))</th>
</tr>
</thead>
<tbody>
<tr>
<td>series path</td>
</tr>
<tr>
<td>parallel path</td>
</tr>
</tbody>
</table>
EXPERIMENTAL ANALYSIS

In order to determine effective conductivity values using the FLOTHERM finite volume model, a full set of measured values is required for a natural convection cooled, vertically oriented, multilayered PCB with various source configurations. The multilayered boards used for these tests consist of a standard FR4 fibreglass-epoxy core with 1 ounce copper laminate on the top and bottom surfaces, as shown in Fig. 2. The dimensions and thermophysical properties of these layers correspond to those presented in Table 1.

Heater modules are constructed of 25mm x 25mm x 10mm 6061-T6 aluminum blocks that are split and hollowed out to contain a foil heater, as shown in Fig. 3. Good thermal contact between the heater and the block is achieved through the use of an aluminum-filled epoxy adhesive. Module temperatures are measured using 5 mil T-type copper-constantan thermocouples embedded in the epoxy layer between the heater and the block. Combined conductive heat losses through the thermocouple wires and the heater leads are typically less than 1.5%.

Heater modules are attached to the circuit board using a thin layer of aluminum-filled epoxy to provide a strong bond and to minimize the contact resistance between the heat source and the board. Three different package distributions are examined in this investigation, as shown in Fig. 4. For each of these three cases, a single centrally located heat source, Fig. 4(a), three horizontal heat sources, Fig. 4(b), and five heat sources, Fig. 4(c), power dissipated by each heater is varied between 1W and 6W per module in 1W steps.

All experimental tests are performed in a vertical flowthrough enclosure designed for natural convection measurements. Ambient and heated air freely enters and exits the enclosure through inlet and outlet plenums, while the test section is protected from unwanted ambient air movement by honeycomb sections at the inlet and outlet. In each test case, the board is suspended in the center of the test section using a polycarbonate card guide assembly. This central location ensures adequate airflow to each side of the board, while the poor contact between the board and the card guide and the low conductivity of the polycarbonate material minimize heat loss from the edges of the board.

Temperature measurement and the control of the power dissipated by the heater modules is performed using a Fluke Helios 1 data acquisition unit connected to an IBM PC. The power supplied to each of the foil heaters is maintained at the prescribed setting by monitoring the voltage and current levels every 300s and setting the power to within ± 0.5% of the intended value over the duration of the test. Steady state conditions are assumed to be achieved when the following three conditions are

![Figure 2: Section of Multilayered Test Board](image)

1 oz. copper
\[ t = 0.038 \text{ mm} \]
\[ k = 386 \text{ W/mK} \]

FR4 - fiberglass/epoxy
\[ t = 1.575 \text{ mm} \]
\[ k = 0.41 \text{ W/mK} \]

![Figure 3: Exploded View of Heater Module](image)

Heater Module
Upper Half

Foil Heater

Epoxy Filled Recess

Heater Module
Lower Half

Heater Module
Figure 4: Heater Configurations for Experimental Test Cases: a) Single Module; b) Three Horizontal Modules; c) Five Modules

satisfied: the temperature change at each thermocouple between successive 300s time intervals is less than 0.1°C; the relative temperature change at each thermocouple between successive time intervals is less than 0.1 %; and the time to reach steady state conditions must be more than $1 \times 10^4$s.

For each of the three configurations and the various power levels, temperatures are measured at each of the component locations and three ambient points surrounding the mounting assembly. Tests are repeated several times in order to ensure consistent, repeatable results. The temperature rise at each of the components for the various configurations and power levels, calculated using the measured temperature minus the mean of the ambient values, are presented in the following sections.

**NUMERICAL ANALYSIS**

The numerical solution of this multilayered PCB problem with various discrete heat source configurations is performed for two reasons. First, the experimental results can be validated using the results of a full numerical model that includes a three layer PCB. As well, using a simplified numerical model with a single layer board of equivalent thickness, the dependence of the effective conductivity on the component density and power level can be determined.

The numerical modeling is performed using FLOOTHERM (1995), a commercially available, finite volume based CFD software package. Each of the cases examined in this work uses a 3D solution domain with laminar, buoyancy-driven flow. All air properties are tabulated values (Incropera & DeWitt, 1990) based on the ambient temperature, $T_a = 20\degree C$. The dimensions and thermophysical properties of the PCB are as presented in Table 1, while the dimensions and locations of the aluminum heater modules are shown in Fig. 4. A thermal conductivity of $k = 180\, W/mK$ is used for the heater module material. All solid portions of the FLOOTHERM model are specified as fully conjugate, such that the model simultaneously solves both the heat conduction in the solid materials and the natural convection in the surroundings.

A number of simplifying assumptions have been made in the formulation of the FLOOTHERM model in order to reduce the size and solution time of the numerical problem. By assuming that a vertical
plane of symmetry divides the board along its center line, the size of the solution domain is reduced to one half. Radiation heat transfer is included in the model on the front and back surfaces of the PCB and on the top surface of each of the heater modules. Emissivities of $\varepsilon = 0.05$ and $\varepsilon = 0.1$ are used for the polished copper laminate and the aluminum, respectively, and the view factor is assumed to be 1 for all cases. Finally, the contact resistance between the heater modules and the PCB will be neglected in this work and perfect thermal contact through the thin layer of epoxy at the interface will be assumed.

In order to capture the highly localized natural convection phenomenon while minimizing the total number of control volumes, a discretization scheme is chosen that concentrates thin control volumes in the regions where boundary layer behavior is anticipated, including the board and heater module surfaces. This control volume grid is refined over several iterations based on the agreement of the full numerical solution for the three layer PCB with the experimental results. Upon achieving grid independence in the numerical results, a comparison of the measured values and the FLOOTHERM solutions for the three layer PCB can be made, as presented in Table 2.

<table>
<thead>
<tr>
<th>$Q$ (W/module)</th>
<th>$\Delta T_{\text{num.}}$ (°C)</th>
<th>$\Delta T_{\text{exp.}}$ (°C)</th>
<th>% diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>CASE 1: One Module</td>
<td>9.28</td>
<td>9.39</td>
<td>-1.2</td>
</tr>
<tr>
<td>1</td>
<td>22.77</td>
<td>25.55</td>
<td>-10.9</td>
</tr>
<tr>
<td>3</td>
<td>35.04</td>
<td>40.72</td>
<td>-13.9</td>
</tr>
<tr>
<td>CASE 2: Three Modules</td>
<td>16.15</td>
<td>15.8</td>
<td>2.2</td>
</tr>
<tr>
<td>1</td>
<td>42.99</td>
<td>41.06</td>
<td>4.7</td>
</tr>
<tr>
<td>3</td>
<td>72.06</td>
<td>74.84</td>
<td>-3.7</td>
</tr>
<tr>
<td>CASE 3: Five Modules</td>
<td>22.52</td>
<td>19.40</td>
<td>16.1</td>
</tr>
<tr>
<td>1</td>
<td>54.07</td>
<td>50.25</td>
<td>7.6</td>
</tr>
<tr>
<td>3</td>
<td>98.70</td>
<td>91.36</td>
<td>8.0</td>
</tr>
</tbody>
</table>

Based on the good agreement between the numerical and measured values, within 10% for most cases, and the repeatability of the experimental results, the numerical model is used to ascertain information regarding $k_{\text{eff}}$. The FLOOTHERM models for each of the three source configurations are modified by replacing the three layer board with a single layer board of equivalent thickness. Numerical values for the center module temperature as a function of the effective conductivity are determined for various power levels, $Q = 1, 3, \text{ and } 6W$ per module, for each of the three configurations shown in Fig. 4. Following a set of tests at the various power levels for a specific source configuration, the value of $k_{\text{eff}}$ used by FLOOTHERM is adjusted and the numerical simulation is repeated. Convergence towards appropriate values of $k_{\text{eff}}$ is determined by comparison of the temperature rise of the center heater module with its corresponding experimental result.
**RESULTS**

Figures 5, 6, and 7 show a comparison between the empirically measured values of temperature rise for the heater module located at the center of the board and the results from the FLOTHERM solutions with a single layer board and various effective thermal conductivities. From these curves it can be seen that the values of the effective conductivity required for good agreement between the numerical and experimental results are not bounded by the upper (parallel) limit described in the theoretical analysis. Instead, \( k_{\text{eff}} \) predicted by this analysis increases dramatically as more packages are added to the board.

In the first test case, where a single heater is placed at the center of the board, the effective conductivity resulting from the FLOTHERM analysis is \( k_{\text{eff}} = 17.4 \text{W/mK} \), which is very close to the parallel limit presented in Table 1, \( k_p = 17.27 \text{W/mK} \). A large value of \( k_{\text{eff}} \) is anticipated for this case based on the small value of the convective heat transfer coefficient \( h \) for natural convection cooling of the board surface. This small \( h \) leads to large values of the fluid-side resistance, \( R_f \), relative to the conduction resistance, \( R \). The heat that enters the PCB from the center component is therefore conducted to the outer regions of the board before being removed by the fluid. This behavior closely resembles that of an annular fin, with the heat flowing radially outward from the source along the board in a parallel fashion.

When three equally sized and powered heater modules are placed on this same board, the effective conductivity predicted by the FLOTHERM analysis increases from that of the first case to \( k_{\text{eff}} = 32.8 \text{W/mK} \). This suggests that, since the fluid side resistance has remained virtually unchanged, a higher \( k_{\text{eff}} \) is required to spread the additional heat further out towards the edges of the board.
Figure 6: Comparison of Experiment with FLOHERM for Various Values of $k_{eff}$ - Case 2: Three Modules

Figure 7: Comparison of Experiment with FLOHERM for Various Values of $k_{eff}$ - Case 3: Five Modules
In the final test case, where a significant portion of the test board’s surface area is covered by five equally sized and powered components, the effective conductivity from the FLOOTHERM single layer board analysis is \( k_{\text{eff}} = 59.7 \text{W/mK} \). Because the fluid side resistance is still relatively unchanged, all additional heat must be conducted to the outer most board area, requiring an extremely large thermal conductivity.

It becomes obvious through this analysis that in a conjugate heat transfer problem such as this, the fluid-side heat transfer cannot be ignored while examining the question of effective thermal conductivity. The resistance to heat flow at the solid-fluid interface can often be the controlling resistance in the heat flow path between the source and the sink, especially for natural convection cases where the thermal boundary layer is thick and the fluid-side resistance is large. If the effect of the fluid-side resistance is considered when estimating \( k_{\text{eff}} \) for a multilayered PCB, the series and parallel limits given in Eqs. (4) and (5) may no longer be the lower and upper bounds on conductivity.

In an effort to better explain the results of this work, a model for the effective conductivity of a multilayered substrate as a function of the number and location of its components is developed. It is proposed that each heat source on the PCB, regardless of its location or power dissipation, has an elliptic shaped effective cooling area that surrounds it, where the heat flow within this effective area resembles that of an annular fin. This elliptic shaped domain extends from the center of the source to either the edge of the board or the adiabatic planes between adjacent sources, located in this case at the midpoints between the equally-heated modules. The schematic in Fig. 8 shows the effective cooling area for the center module for each of the three test cases, while Table 3 presents the major and minor axis lengths, \( a \) and \( b \), for each of the elliptic areas of influence.

In order to utilize this effective cooling area in the calculation of effective conductivity, the elliptical domain is approximated using a circular region of equivalent surface area. The radius of this circle is defined as the effective radius of influence, \( r_{\text{eff}} \), a measure of the size of the conduction path surrounding a particular component on the board. The effective radius is related to the dimensions of the elliptic region by:

\[
r_{\text{eff}} = \sqrt{\frac{ab}{4}}
\]

(6)

where the results of this calculation for the various module configurations are presented in Table 3. From the general definition of the conduction resistance:

\[
R = \frac{L}{kA}
\]

(7)

Figure 8: Schematics of Effective Cooling Areas for Various Module Configurations
Table 3: Dimensions of the Elliptic Area of Influence and the Effective Radius of Influence for Various Module Configurations

<table>
<thead>
<tr>
<th>CASE</th>
<th>(a) (m)</th>
<th>(b) (m)</th>
<th>(\tau_{eff}) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.15</td>
<td>0.15</td>
<td>0.075</td>
</tr>
<tr>
<td>2</td>
<td>0.0375</td>
<td>0.15</td>
<td>0.0375</td>
</tr>
<tr>
<td>3</td>
<td>0.0375</td>
<td>0.05</td>
<td>0.0217</td>
</tr>
</tbody>
</table>

An expression for the effective thermal conductivity can be formulated:

\[
k_{eff} = \frac{1}{R} \left( \frac{L}{A} \right)
\]  

(8)

Since it is convenient to assume that \(R\) can be calculated independent of source strength or position, the geometric mean of the series and parallel limits, Eqs. (2) and (3), is used:

\[
R = \sqrt{R_s \cdot R_p}
\]  

(9)

The remaining term in Eq. (8), the ratio of the path length and cross sectional area, is equated to the effective radius, determined using Eq. (6) and the elliptical regions shown in Fig. 9:

\[
\left( \frac{L}{A} \right) = \frac{1}{\tau_{eff}}
\]  

(10)

The resulting expression for the effective conductivity is:

\[
k_{eff} = \frac{1}{R \cdot \tau_{eff}} \cdot \frac{h_s}{h_p}
\]  

(11)

where \(h_s\) and \(h_p\) are series and parallel conductances. This conductance ratio can be calculated using:

\[
\frac{h_s}{h_p} = \frac{k_s}{k_p} \frac{L}{\sum_{i=1}^{N} t_i}
\]  

(12)

where the series and parallel conductivity limits, \(k_s\) and \(k_p\), are determined using Eqs. (4) and (5). This ratio of the conductances is a fixed value for a given board, independent of component power or placement.

Table 4 compares the effective thermal conductivity predicted by the model with the results of the FLOOTHERM analysis and it shows the excellent agreement between the model and the numerical values for each of the three configurations tested in this work.
Table 4: Comparison of Model Predictions for Effective Thermal Conductivity with Numerical Results

<table>
<thead>
<tr>
<th>CASE</th>
<th>$k_{eff}$ model (W/mK)</th>
<th>$k_{eff}$ num. (W/mK)</th>
<th>% diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.4</td>
<td>17.4</td>
<td>-0.2</td>
</tr>
<tr>
<td>2</td>
<td>34.8</td>
<td>32.8</td>
<td>5.7</td>
</tr>
<tr>
<td>3</td>
<td>59.7</td>
<td>60.3</td>
<td>1.0</td>
</tr>
</tbody>
</table>

SUMMARY AND CONCLUSIONS

Experimental and numerical techniques have been used to investigate heat conduction in multilayered printed circuit boards as a function of the number, location, and power dissipation of surface mounted components. By matching the measured component temperatures with the results of the FLOHERM analysis, effective thermal conductivities have been determined which allow the complex multilayered board to be modeled using a single layer. The relationship between $k_{eff}$ and the component density for the specific case of vertically oriented, natural convection cooled PCBs has been successfully modeled by introducing the effective radius of influence, $r_{eff}$, defined by the component locations and board dimensions. Once the effective radius is established, the length of the heat flow path in the effective conductivity calculation is restricted to the radius of influence of each heat source and not the overall dimensions of the laminates in the PCB. As a result, the portion of the board surrounding each heat source will have a unique effective thermal conductivity, based on its position and power relative to its neighbors.

As discussed previously, the effective conductivity values presented in this work greatly exceed the series and parallel quantities that were previously thought of as the lower and upper bounds. Although some combination of these values may be effective for predicting $k_{eff}$ for pure conduction problems, in this natural convection case the heat transfer has been shown to be dominated by the relatively small values of the convective heat transfer coefficient, $h$. When these convection effects are taken into account, it is clear that the values presented in Table 1 are no longer the lower and upper bounds on the effective conductivity.

It should be made clear that the results and model presented in this work are specific to this particular test case. Additional work is required involving different source configurations, different board layer configurations, unequal component power levels, and mixed and forced convection cooling in order to develop a more fundamental understanding of the factors influencing the effective thermal conductivity. However, many of the observations made concerning the behavior of $k_{eff}$ for various configurations and conditions can be useful to researchers doing numerical or analytical modeling of multilayered PCBs.

REFERENCES

