A COMPACT MODEL FOR SPHERICAL ROUGH CONTACTS

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OVERVIEW

• Introduction
• Objectives
• Present Model
• General Pressure Distribution
• Dimensional Analysis
• Comparison with Experimental Data
• Conclusions
INTRODUCTION

Contact of Spheres

• elastic, smooth
  – Hertz (1881) theory of elastic contact of spheres
    \[ P_{Hz} \frac{\dot{r}}{a_{Hz}} \propto \sqrt{1 + \frac{\dot{r}^2}{a_{Hz}^2}} \]

• elastic, rough spheres, elastic microcontacts
  – Greenwood and Tripp (1967)

• plastic, rough surfaces, plastic microcontacts
  – Mikic and Roca (1974)

• roughness parameter
  – Greenwood et al. (1984)

\[ \alpha = \frac{\sigma \rho}{a_H^2} = \sigma \left( \frac{16 \rho E'^2}{9 F^2} \right)^{1/3} \]
OBJECTIVES

• develop model to predict the parameters of spherical contact:
  – pressure distribution,
  – elastic deformation,
  – compliance,
  – number of microcontacts,
  – size of the contact area

• derive simple correlations for determining contact parameters that can be used in other analyses such as thermal contact models
conforming rough contacts

Plastic Model

- Gaussian surfaces
- plastically deformed hemispherical asperities
- cross-level theory

\[
\frac{A_r}{A_a} = \frac{1}{2} \text{erfc} \ R
\]

\[
a_s = \sqrt{\frac{8}{2}} \frac{\ddot{Y}}{m} \text{exp} \dot{Y} R^2 \text{erfc} \ R
\]

\[
n_s = \frac{1}{16} \frac{\ddot{Y}}{m} \dot{\sigma} \left( \frac{\exp\left( \frac{1}{2} R^2 \right)}{\text{erfc} \ R} \right) A_a
\]
Hegazy (1985)

- microhardness may not be constant throughout the material as a result of machining process
- microhardness decreases with increasing depth of indentation until bulk hardness level

\[ H_v : c_1 \dot{Y}d_v^r \sigma^2 \]

\[ d_v^r : d_v/d_0 \]
PRESENT MODEL: ASSUMPTIONS

- spherical surfaces
- Gaussian asperity distribution
- plastic microcontacts
- elastic macrocontact

\[ Y(r) = \omega_b(r) - u(r) = \omega_b(r) - u_0 + \frac{r^2}{2\rho} \]
ASSUMPTIONS 2

- deformation of each asperity is independent of its neighbors
- no friction
- first loading cycle
- static contact
RELATIONSHIPS FOR MECHANICAL MODEL

\[ \lambda(r) = \frac{Y(r)}{\sqrt{2\sigma}} \]

\[ P(r) = \frac{1}{2} H_{mic}(r) \operatorname{erfc}(\lambda(r)) \]

\[
\omega_b(r) = \begin{cases} 
\frac{2}{E^*} \int_0^\infty P(s) \, ds & r = 0 \\
\frac{4}{\pi E^*} \int_0^r P(s) K\left(\frac{s}{r}\right) \, ds & r > s \\
\frac{4}{\pi E^*} \int_r^\infty P(s) K\left(\frac{r}{s}\right) \, ds & r < s 
\end{cases}
\]

\[ F = 2\pi \int_0^\infty P(r) \, r \, dr \]

\[ a_s(r) = \sqrt{\frac{8}{\pi} \left(\frac{\sigma}{m}\right)} \exp\left(\lambda^2(r)\right) \operatorname{erfc}(\lambda(r)) \]

\[ H_{mic}(r) = c_1 \left(\sqrt{2\pi a_s(r)}\right)^{c_2} \]
NUMERICAL ALGORITHM – INSIDE LOOP

From Main Loop $\mu_0$

Calculate $P(r)$

Calculate $\omega_{b,new}(r)$

Calculate $P_{new}(r)$

$P_{new}(r) - P(r) \over P_{new}(new) \cdot TOL.$

$P(r) = P_{new}(r)$

$\omega_b(r) = \omega_{b,new}(r)$

Not Acceptable

Acceptable
SUCCESSIONAL ITERATION

\[ F^* = \frac{F - F_{\text{Calculated}}}{F_{\text{Calculated}}} \]

Point 1

Point 2

Line passing through points 1, 2

\( u_{0,new} \)

\( u_{0,1} \)

\( u_{0,2} \)
### OUTPUT PARAMETERS OF MODEL

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of curvature</td>
<td>$\rho = 25 \text{ (mm)}$</td>
</tr>
<tr>
<td>Surface slope</td>
<td>$m = 0.107$</td>
</tr>
<tr>
<td>Roughness</td>
<td>$\sigma = 1.414 \text{ (\mu m)}$</td>
</tr>
<tr>
<td>Young’s modulus $E_1$</td>
<td>$E_2 = 204 \text{ (GPa)}$</td>
</tr>
<tr>
<td>Poisson’s ratio $\nu_1$</td>
<td>$\nu_2 = 0.3$</td>
</tr>
<tr>
<td>Force $F$</td>
<td>$= 50 \text{ (N)}$</td>
</tr>
<tr>
<td>Microhardness $c_1$</td>
<td>$= 6.27 \text{ (GPa)}$</td>
</tr>
<tr>
<td>Microhardness $c_2$</td>
<td>$= -0.15$</td>
</tr>
<tr>
<td>Sample dia. $b_L$</td>
<td>$= 25 \text{ (mm)}$</td>
</tr>
</tbody>
</table>

#### Diagrams

**a)** Non-Dimensional Pressure Distribution

**b)** Microcontacts Radius ($\mu$m)

**c)** Microcontacts Density ($m^2$)

**d)** Microhardness (GPa)
GENERAL PRESSURE DISTRIBUTION

\[ P(\xi) = P_0 \left(1 - \xi^2 \right)^\gamma \]

\[ \gamma = 1.5 \frac{P_0}{P_{0,H}} \left( \frac{a_L}{a_H} \right)^2 - 1 \]

\[ P_0 = \left(1 + \gamma\right) \frac{F}{\pi a_L^2} \]

Hertzian limit

\[ P_H \left( \frac{r}{a_H} \right) = P_{0,H} \left(1 - \left( \frac{r}{a_H} \right)^2 \right)^\gamma \]

\[ \gamma_H = 0.5 \]

\[ P_{0,H} = \frac{1.5F}{\pi a_H^2} \]
DIMENSIONAL ANALYSIS

- effective microhardness,
  $$H_{mic} = \text{Const.}$$

- surface slope $m$ is assumed to be a function of surface roughness, Lambert (1995)
  $$m : 0.076 \rightleftharpoons 0.52$$

- maximum contact pressure is a function of
  $$P_0 : P_0 \left[ , \right], E^r, F, H_{mic} \downarrow$$

- three non-dimensional parameters

<table>
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<th>Parameter</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective elastic modulus, $E^r$</td>
<td>$ML^{1}T^{2}$</td>
</tr>
<tr>
<td>Force, $F$</td>
<td>$MLT^{2}$</td>
</tr>
<tr>
<td>Microhardness, $H_{mic}$</td>
<td>$ML^{1}T^{2}$</td>
</tr>
<tr>
<td>Radius of curvature, $\left[ \right]$</td>
<td>$M$</td>
</tr>
<tr>
<td>Roughness, $\left[ \right]$</td>
<td>$M$</td>
</tr>
<tr>
<td>Max. contact pressure, $P_0$</td>
<td>$ML^{1}T^{2}$</td>
</tr>
</tbody>
</table>

$$\alpha = \frac{\sigma}{\omega_{0,H}} \equiv \frac{\sigma \rho}{a_H^2}$$

$$\tau = \frac{\rho}{a_H}$$

$$E^D : \frac{E^r}{H_{mic}}$$
EFFECT OF MICROHARDNESS PARAMETER

effect of microhardness parameter on the maximum contact pressure is small and therefore ignored.
CORRELATIONS

\[ P_0' = \frac{P_0}{P_{0,H}} = \frac{1}{1+1.37 \alpha / \tau^{0.075}} \]

\[ a_L' = \frac{a_L}{a_H} = \begin{cases} 
1.605 / \sqrt{P_0'} & 0.01 \leq P_0' \leq 0.47 \\
3.51 - 2.51P_0' & 0.47 \leq P_0' \leq 1 
\end{cases} \]
Greenwood and Tripp (1967) disadvantages:

- complex, requires computer programming and numerically intensive solutions
- $\beta$ and $\eta_s$ cannot be measured directly, sensitive to the surface measurements
- constant summit radius $\beta$ is unrealistic
Elastic Deformation of Half-Space

- using general pressure distribution, relationships are derived for:
  - elastic deformation of half-space
  - compact correlation is derived for compliance
EXPERIMENTAL DATA

\[ \tau = \frac{\rho}{a_H} \]

- **Kagami, Yamada, and Hatazawa (KYH) 1982**
  \[ 16.8 \leq \tau \leq 187.4 \]
  \[ \rho = 3.15 \text{ mm}, \ 0.082 \leq \sigma \leq 1.45 \mu \text{m}, \ 0.19 \leq F \leq 88 \text{ N} \]
  carbon steel spheres – carbon steel and copper flats

- **Greenwood, Johnson, and Matsubara (GJM) 1984**
  \[ 31 \leq \tau \leq 170.8 \]
  \[ \rho = 12.7 \text{ mm}, \ 0.19 \leq \sigma \leq 2.2 \mu \text{m}, \ 4.8 \leq F \leq 779 \text{ N} \]
  hard steel balls – hard steel flats

- **Tsukada and Anno (TA) 1979**
  \[ 7.8 \leq \tau \leq 47.6 \]
  \[ \rho = 1.5, 5, 10 \text{ mm}, \ 0.11 \leq \sigma \leq 2.1 \mu \text{m}, \ 23.5 \leq F \leq 1375 \text{ N} \]
  SUJ 2 spheres – SK 3 flats

\[ \alpha = \sigma \rho / a_H^2 \]
COMPARISON WITH DATA: CONTACT RADIUS

Tsukada and Anno 1979, specimens: SUJ 2 spheres and SK 3 flats

Test: TA1 TA2 TA3 TA4 TA5 TA6 TA7 TA8
\( \sigma \) mm 0.11 0.35 0.84 0.11 0.35 0.35 0.12 0.35
\( \rho \) mm 1.5 1.5 1.5 5 5 5 10 10

Kagami, Yamada, and Hatazawa 1982

Test: KYH1 KYH2 KYH3 KYH4
\( \sigma \) mm 0.457 0.180 1.45 0.457
specimens: steel spheres of radius 3.18 mm
KYH 1&2: carbon steel (0.3% C) flats
KYH 3&4: pure copper (99.9% pure) flats

Greenwood, Johnson, and Matsubara 1984

Test: GJM1 GJM2 GJM3 GJM4
\( \sigma \) mm 0.19 0.54 1.7 2.2
specimens: hard steel balls of radius 12.7 mm
and hard steel flats

\( a'_L = a_L / a_H \)

\( P'_0 = P_0 / P_{0,H} \)

RMS difference 6.2%.

more than 160 points
COMPARISON WITH DATA: COMPLIANCE

\[
\kappa' = \frac{\kappa}{\kappa_H} = 0.5 \left( \alpha_L' \right)^2 + \frac{8P_0' \alpha_L'}{\pi^2 \left[ 4.79 - 3.17 \left( P_0' \right)^{3.13} \right]}
\]

Kagami, Yamada, and Hatazawa 1982

test KYH5 KYH6 KYH7 KYH8

<table>
<thead>
<tr>
<th>σ (μm)</th>
<th>1.45</th>
<th>0.082</th>
<th>0.457</th>
<th>0.082</th>
</tr>
</thead>
</table>

specimens: steel spheres of radius 3.18 mm.
KYH 5&6: carbon steel (0.3% C) flats
KYH 7&8: pure copper (99.9% pure) flats

RMS difference 7.7%.

more than 40 points
SUMMARY AND CONCLUSIONS

• a general pressure distribution that encompasses all spherical rough contacts including Hertzian limit is proposed

• compact correlations for contact radius and compliance are proposed and validated with experimental data

• It is shown that the non-dimensional maximum contact pressure is the main parameter that controls the solution of spherical rough contacts
ACKNOWLEDGMENTS

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